



From the physics of classical pulsators to the distance scale using high resolution spectroscopy and interferometry

N. Nardetto¹

Laboratoire Lagrange, UMR7293, Univ. de Nice Sophia-Antipolis, CNRS, Obs.de la Côte d'Azur, France
e-mail: Nicolas.Nardetto@oca.eu

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Abstract. In this review, we will illustrate how high resolution spectroscopy and interferometry are powerful tools to probe the physics of Classical pulsators, and in particular Cepheids, from their interior up to their exterior: photosphere and limb-darkening, dynamical structure of the atmosphere, chromosphere, and circumstellar environment. A better understanding of the physics of Cepheids is a requirement to improve the Baade-Wesselink (BW) method of distance determination (and in particular its projection factor) in order to open a new route toward H_0 . A key aspect of this work is also to test the universality of the physics of Cepheids and possibly unveil some systematics in the period-luminosity of Cepheids.

Key words. Stars: Variables: RR Lyrae stars, Anomalous Cepheids – Dwarf galaxies: Resolved stellar populations

1. Introduction

Classical Cepheids are yellow giant and supergiant pulsating stars used as stellar candles in the universe up to 40 Mpc and beyond with the recent launch of the James Webb Space Telescope (Yuan et al. 2022). The relation between their mean absolute magnitude and the logarithm of their pulsation period, first discovered by Leavitt & Pickering (1912), is currently used to calibrate the Type Ia supernovae luminosity relation and the expansion rate of the universe (i.e. the Hubble-Lemaître constant, H_0). Several years ago, Riess et al.

(2016) used four anchors in order to calibrate the period-luminosity (PL) relation: the megamaser NGC4258 (Humphreys et al. 2013), the distance to LMC from detached eclipsing binaries (Pietrzyński et al. 2013), 15 parallaxes of Galactic Cepheids (GCs) from Hubble Space Telescope (HST), and the distance to M31 and M33 from detached eclipsing binaries (DEBs) (Ribas et al. 2005; Vilardell et al. 2010). They obtained a value of H_0 with a precision of 2.4%. Later, the precision was improved first by adding the HST parallaxes of 7 long-period GCs (Riess et al. 2018), and then by considering the improved distance determination of LMC to 1% from DEBs, which reduced the

precision on H_0 to 1.9%. Interestingly, Breuval et al. (2020) used the Gaia DR2 parallaxes of the companion of Cepheids, as found by (Kervella et al. 2019a,b), and of open clusters to constrain the Hubble Constant and derive a value of H_0 consistent with previous results and precise to 2.6%. One year later, the SHOES team used the parallaxes of 75 GCs from Gaia DR3 to reach a 1.8% precision on H_0 (Riess et al. 2021) and just after, they increased the number of anchor galaxies with SNIa from 19 to 42 yielding to an unprecedented precision on H_0 of 1.4% (Riess et al. 2022).

The metallicity dependence of the PL relation of Cepheids is surely one of the key issues as it can affect H_0 up to half of the error budget as mentioned by Romaniello et al. (2022). The calibration of the period-luminosity-metallicity relation is thus currently deeply studied by the community (Groenewegen 2018; Proxauf et al. 2018; da Silva et al. 2022; Ripepi et al. 2020, 2022a; Lemasle et al. 2017; Romaniello et al. 2022; Gieren et al. 2018; Storm et al. 2011a,b; Gilligan et al. 2021; Breuval et al. 2021; De Somma et al. 2021) not only for the distance scale but also for galactic archeology (Deb et al. 2019; Ripepi et al. 2022b; Inno et al. 2019; Minniti et al. 2021; Prudil et al. 2021) to cite only some examples. Another way to confirm or infirm the Hubble tension is to use different roads towards H_0 using different kinds of classical pulsators, like miras (Huang et al. 2020) or the combined Type 2 Cepheids and RR Lyrae period luminosity relation (Rich et al. 2018; Braga et al. 2020).

A more difficult route toward H_0 is the use of the Baade-Wesselink method of distance determination. This method, based on spectroscopy and interferometry, has the main advantage to allow an individual distance determination of Cepheids in the Local Group. With the future Extremely Large Telescope (ELT) it will be indeed possible to provide new anchors to the calibration of the PL relation of Cepheids opening a new route toward H_0 . The second advantage of the method is that it requires a complete view of the physics of Cepheids from their pulsating envelope up to their environ-

ment. This is the first step to test the universality of the physics of Cepheids and possibly unveil some systematics in the period-luminosity of Cepheids.

2. The Baade-Wesselink method and the projection factor

The Baade-Wesselink (BW) method was first described by Lindemann (1918), then later extended by Baade (1926) and Wesselink (1946). For almost a century, the BW method was used to derive the distance of Cepheids. The concept is simple: distances are computed using measurements of the angular diameter over the whole pulsation period along with the stellar radius variations deduced from the integration of the pulsation velocity V_{puls} . The latter is linked to the observed radial velocity (RV) by the projection factor $p = V_{\text{puls}}/RV$ (Hindsley & Bell 1986; Nardetto et al. 2004, 2009). There are basically three versions of the BW method corresponding to different ways of determining the angular diameter curve: a photometric version based on the Surface-Brightness Color Relation (SBCR) (Fouque & Gieren 1997; Fouqué et al. 2007; Storm et al. 2011a,b), an interferometric version (Lane et al. 2000; Kervella et al. 2004b; Mérand et al. 2005), and a more recent one which combines several photometric bands, velocimetry and interferometry (SPIPS; Mérand et al. 2015).

In principle, the BW method can be applied to any kind of pulsating star with the condition of radial pulsation. It is indeed basically a parallax measurement of the amplitude of the pulsation of the star. Recently, it has been applied to a sample of RR Lyrae stars (Muraveva et al. 2015) and even to the prototype RR Lyrae itself (Jurcsik et al. 2017). As RR Lyrae experiences a Blazkho modulation effect, the application of the method leads to several complications. The BW method was also applied successfully to δ Scuti (Nardetto et al. 2014) and β Cephei stars (Nardetto et al. 2013), as well as Miras (Lacour et al. 2009).

However, the BW method is currently not used in the distance scale calibration. Indeed, more than ten years ago, Riess et al. (2009) wrote: *We have not made used of additional*

distance measures to Galactic Cepheids based on the BW [...] as they are much more uncertain than well-measures parallaxes, and [...] appear to be under refinement due to uncertainties in their projection factors, as discussed in (Fouque & Gieren 1997) and (van Leeuwen et al. 2007). Recently, a study has shown that the projection factors of Cepheids are indeed highly dispersed (even for Cepheids with the same period), which limits the precision of the BW method to 5-10% (Trahin et al. 2021; Gallenne et al. 2017). In the following, we explore the different sources of uncertainties of the BW method as well as the physics of Cepheids, and see how interferometry and spectroscopy are useful in this domain.

3. The physics of the projection factor: from the atmosphere to the photosphere

As already mentioned the projection factor is used to convert the radial velocity into the pulsation velocity of the star. For a Cepheid described simply by a uniform disk pulsating, its value is 1.5 (whatever the pulsation phase). But actually, the radial velocity of each surface element of the star is projected along the light of sight and weighted by the intensity distribution of the Cepheid. The limb-darkening of δ Cep reduces the p-factor significantly, and the so-called geometric projection factor (p_0 , **step 1** in Fig. 1) is between 1.36 to 1.39, depending on the wavelength in the visible range (Getting 1934; Nardetto et al. 2006b; Neilson et al. 2012). The time variation of the p-factor, due mainly to limb-darkening variation, is neglected as it has no impact on the distance Nardetto et al. (2006a). However, a Cepheid is not simply a limb-darkened pulsating photosphere, it has also an extended atmosphere with various spectral lines (in absorption) forming at different levels from which we derive the radial velocity curve used in the BW method. Moreover, there is a velocity gradient in the atmosphere of the Cepheid, which can be measured from spectroscopic observations (**step 2** in Fig. 1). Then, depending on the line considered, the amplitude of the radial velocity curve will not be the same and the resulting

projection factor will be different. In Fig. 1 (f_{grad} , **step 3**), we show the impact of the atmospheric velocity gradient on the p-factor for a line forming rather close to the photosphere (line depth of about 0.1). The higher the line forming region is in the atmosphere, the lower the projection factor is (up to 3% compared to p_0 in the case of δ Cep). The last correction on the projection factor ($f_{\text{o-g}}$, **step 4**) is more subtle. In spectroscopy, the radial velocity is actually a velocity associated with the moving gas in the line forming region, while in photometry or interferometry, we probe an optical layer corresponding to the black body continuum (i.e. the layer from which escape the photons). Therefore, an additional correction on the projection factor of several percents (independent of the wavelength or the line considered) has to be considered. A relation between the period of Cepheids and the p-factor has been established using this approach for a specific line Nardetto et al. (2007) or using the cross-correlation method Nardetto et al. (2009).

At the moment, the core of the decomposition of the projection seems to be confirmed as the velocity gradient measured by HARPS-N for δ Cep is consistent to first order with hydrodynamical calculations (Nardetto et al. 2017), even if a rescaling term of velocity amplitudes is necessary because the lack of convection in the model. Also, the inverse BW projection factor (about 1.25) derived from the distance of the star provided by Majaess et al. (2012) is consistent with the one-dimensional hydrodynamical model of Nardetto et al. (2004) and the 2-dimensional time-dependent convective model of Vasilyev et al. (2017, 2018). It is also consistent with another totally independent method of determination of the projection factor based on eclipsing binaries (Pilecki et al. 2013). There is thus a kind of convergence among these different approaches. However, the dispersion of the period-projection factor relation seen in Trahin et al. (2021) and Gallenne et al. (2017) tells us that something is however still missing. One possibility is that the extrapolation to the photosphere is not consistent. This could be due to the use of the cross-correlation method in order to de-

rive the radial velocity. Indeed, this method is sensitive to stellar rotation, line width, and has the drawback of mixing thousands of lines forming at different levels in the atmosphere. Moreover, the dynamical structure of the atmosphere might be more complex than expected as shown by Nardetto et al. (2018) who found a shift between optical and infrared spectroscopic lines using the CRIRES spectrograph. On top of this, one should not forget also that cycle-to-cycle variations have been found in long-period Cepheids (Anderson 2014, 2016; Anderson et al. 2016). In this context, ideally, one would need simultaneous optical and near-infrared spectroscopic observation of Cepheids over several consecutive cycles in order to improve our knowledge of the dynamical structure of the atmosphere of Cepheid.

Another source of uncertainty of the BW method, rather on the photospheric side could be the SBCR. The SBCR is usually calibrated using interferometric measurements and depends on the luminosity class (Salsi et al. 2021, 2022). Thus, in order apply it to Cepheids, one has to consider a SBCR based on the interferometric observations of giant and/or supergiant stars (for instance di Benedetto (1993); Groenewegen (2004); Salsi et al. (2021)) or even better based on observations of Cepheids themselves (Fouque & Gieren 1997; Kervella et al. 2004a). From recent analysis (Nardetto et al. 2022, submitted), the impact of the SBCR on the projection factor can be up to 8%. A large set of homogeneous and precise interferometric and photometric measurements is thus necessary to calibrate an accurate and consistent SBCR dedicated to Cepheids. Interestingly, SBCRs are also used for distance determination of eclipsing binaries (Pietrzyński et al. 2019; Graczyk et al. 2020) and for the characterization of exoplanet host stars in the context of the PLATO space mission (Gent et al. 2022). It is timely to improve the calibration of such relations.

4. The chromosphere and the Close CircumStellar Environment (CSE) of Cepheids

The physical nature of the pulsating chromosphere of Cepheids and their close environment is complex and might leak into the projection factor uncertainties when applying the BW method. These can also affect actually the PL relation of Cepheids.

In order to probe the dynamical structure of the chromosphere, one has to study specific lines such as Paschen, $H\alpha$ or calcium triplet (Nardetto et al. 2006a; Wallerstein et al. 2015, 2019; Hocdé et al. 2020). Interestingly, Engle et al. (2017); Evans & Engle (2019) found x-ray (resp. UV) emission peaks at maximum (resp. minimum) radius of δ Cep. These X-ray emission can be explained theoretically by pulsation driven shocks (Moschou et al. 2020). However, curiously, no X-ray emission is found for η Aql (Evans et al. 2021). Cepheids can have strong compression waves in the atmosphere or shocks (seen generally in chromosphere lines), but this is also the case of RR Lyrae (Gillet et al. 2017, 2019; Duan et al. 2021; Preston et al. 2022). In this framework, the atypical Cepheid X Sgr with its line doubling remains a mystery, even if the double shocks remain a good hypothesis (Mathias et al. 2006).

Besides, the CircumStellar Environnement (CSE) of Cepheids was discovered by interferometry several years ago (Kervella et al. 2006). Since then, the CSE (of Cepheids) has been characterized using either interferometry (Mérand et al. 2006; Gallenne et al. 2013a; Nardetto et al. 2016), imaging (Gallenne et al. 2011, 2012) or spectral energy distribution analysis (Groenewegen & Jurkovic 2017; Groenewegen 2020; Gallenne et al. 2021). A summary of these observations can be found in Nardetto (2018) (Table 3.6).

An effort has been made recently to observe the CSE of Cepheids in unexplored domains of wavelength. First, δ Cep was observed with the VEGA/CHARA instrument Mourard et al. (2009) and an unexpected visible CSE contributing to 7 ± 1 % to the total flux, with a size of 8.9 ± 3.0 mas was discovered

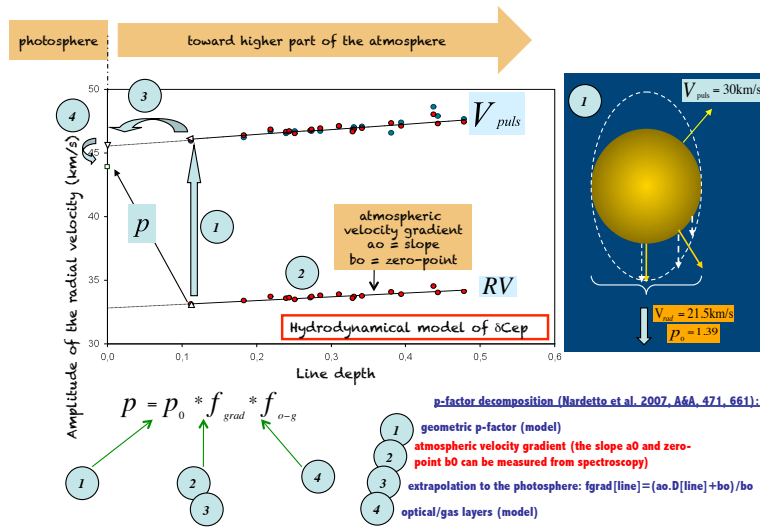


Fig. 1. The p-factor decomposition is illustrated based on the hydrodynamical model of δ Cep (see Nardetto et al. (2007)). The different steps are explained in Sect. 3.

(but a visible background filling the field of view of the interferometer is not excluded; see Nardetto et al. (2016)). Using VLTI/MATISSE observations, Hocdé et al. (2021) found for ℓ Car a CSE of 2 stellar radii with a flux contribution of 7% in the L band. Besides, very interestingly no dust signature was found. Instead, Hocdé et al. (2020) developed an analytical model of a compact CSE (15% of the radius), composed of ionized gas, in order to reproduce the infrared excess of Cepheids as derived from the SPIPS approach. These models do not reproduce the typical size found for the CSE by interferometry (typically 2-3 stellar radii), but some improvements are foreseen. Characterizing and understanding the CSE of Cepheids with finesse is crucial. Indeed, as noted by Efstathiou (2014), if Cepheids from SHOES project are too bright by 0.1 magnitude, the Hubble tension is resolved. And the typical impact of CSE on the magnitude in K (and even in V) can be of 0.05 to 0.1 magnitudes from the first studies (Hocdé et al. 2020). Thus, if Cepheids are not universal and in particular, if they have a different CSE in local and distant galaxies it could impact the PL relation and H_0 . Moreover, they have also an impact in the application of the BW method and possibly the projection factor. It is thus of high importance to better characterize the CSE of Cepheids within the instability strip. Interestingly, CSEs

are also found around different kinds of classical pulsators such as Miras stars. For instance, interferometry helped also a lot to characterize the environment of Mira stars (Haubois et al. 2015; Wittkowski et al. 2016; Khouri et al. 2018; Paladini et al. 2012) and many studies aimed at probing their dynamical structure (Lèbre et al. 2014; Kravchenko et al. 2020). It could be of high interest to compare the properties of such objects.

5. The binarity of Cepheids

80% of Cepheids are binaries (Kervella et al. 2019b,a). In this context, interferometry is an extremely powerful tool to study the properties of the companions of Cepheids (Gallenne et al. 2013b, 2014, 2016, 2019). Using CHARA/MIRC Gallenne et al. (2018) could derive the best precision ever of the distance of a Cepheid: about 1% on the distance of V1334 Cyg. Recently, Karczmarek et al. (2022) have performed a population synthesis of binary Cepheids and evaluated impact of binarity on the distance scale. They found that the impact on the PL relation will depend on the fraction of binarity (1-100%) in anchors galaxies (MW, LMC, SMC, ...) and in distant galaxies. But in any case, the effect in terms of magnitude is negligible even if binaries can potentially impact also the dispersion of the

PL relation. Besides, strong efforts are done to study the binarity of Cepheids and RR Lyrae (Hajdu et al. 2021, 2022; Pilecki et al. 2021; Salinas et al. 2020).

6. Conclusion

At the moment, the BW method of distance determination is locked because of the projection factor issue. This is true whatever the BW version used: interferometry, SBCR or even a combination of both (SPIPS). To unlock it, several aspects should be considered:

1. The method used to derive the radial velocity curve is a key aspect as it can change the distance by about 10%. Importantly, the centroid method should be preferred as it is independent of the rotation and the FWHM of the line.
2. Characterizing better the pulsating atmosphere of Cepheids is fundamental. The atmospheric velocity gradient can indeed break the consistency between the interferometric/photometric measurements on one side and the spectroscopic measurements on the other side.
3. Checking the cycle to cycle variations of Cepheids is also important as non synchronous photometric, spectroscopic and interferometric observations could potentially alter the distance determination.
4. The choice of the SBCR is crucial as difference of 8% can be obtained on the distance (Nardetto et al. 2022, submitted). The disagreements between SBCRs in the literature need clarification even if their origin is probably due to the use of different sample of stars, methods and instruments. One should use a dedicated SBCR calibrated using homogeneous photometric and interferometric measurements. And the same photometric system should be used when applying the SBCR.
5. The presence of a CSE can alterate the SBCR version of the BW method, but also the interferometric or SPIPS ones, if it is not taken into account. This CSE can indeed create an offset in the visible or infrared magnitudes or even both. The CSE

of Cepheids in the instability strip should be studied, characterized and parametrized in the calibration/use of the SBCR.

In summary, studying the physics of Cepheids is fundamental to unlock the BW method of Cepheids and open a new road to the Hubble constant. It could also help to refine the period-luminosity relation and resolve/confirm the Hubble tension.

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