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Latest advances in theoretical models for classical pulsators

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Abstract. Classical pulsating stars are important distance indicators and stellar population tracers. On this basis a comprehensive investigation of their properties based on both an observational and a theoretical approach is crucial.
The most important theoretical tools and results obtained for both classical Cepheids and RR Lyrae are outlined and their reliability is tested against observed pulsation properties. The dependence of predicted properties on several input model parameters and assumptions is discussed with a final mention of possible future developments.

1. Introduction

In the context of current and future astrometric, photometric, and spectroscopic surveys, the study of radially pulsating stars is crucial for several aspects of stellar and galactic astrophysics. Indeed these objects have the following key advantages: i) thanks to the cyclic or periodic variability of their magnitude they are easy to identify; ii) the period and the amplitude of the oscillation are directly measurable from time-series observations, being independent of both distance and reddening uncertainties; iii) the period of the oscillation is well known to be anti-correlated with the stellar mean density and the combination of this relation with the Stefan-Boltzmann law provides a relation connecting the period to the mass, the luminosity and the effective temperature (PLMT relation). The possibility to relate a pulsation property such as the period (in some cases this also holds for the amplitude of the oscillation, see e.g. Bono et al., 2007, and references therein)) to evolutionary parameters that in turn depend on the age and the chemical composition, has, since van Albada & Baker (1971), opened a new dimension in the investigation of stellar populations. Moreover, in the case of Classical Cepheids the prediction of a Mass-Luminosity (-chemical abundances) relation for intermediate mass (from ~ 3 to $\sim 13\ M_{\odot}$) stars in the central Helium burning phase, allows us to cancel the mass dependence in the PMLT relation and derive period-temperature-luminosity relations that in the observational space correspond to period-magnitude-color (PLC) relations that are traditionally used, together with the simpler but less precise period-magnitude (PL) relations to infer individual and mean distances of extragalactic Cepheid samples (see e.g. Freedman et al., 2001; Sandage et al., 2004; Riess et al., 2022, and references therein). In general, the relations connecting the period to the intrinsic properties of the stars, make pulsating stars excellent stellar population tracers and distance indicators. In this context, the observational investigations need to be combined with theoretical studies in order to understand and quantify how the dependence of pulsation on intrinsic stellar properties can affect the observed behaviours. For example, several theoretical and observational efforts have been performed in the last few decades to understand the level of universality of adopted PL and PLC relations (see e.g. Breuval et al., 2022; De Somma et al., 2022; Ripepi et al., 2022, and references therein). In the following, current theoretical approaches to the study of stellar pulsation are presented, together with some relevant results for the most famous classes of pulsating stars adopted in the literature as distance indicators and stellar population tracers, namely Cepheids and RR Lyrae.

2. Current approaches to the modelling of pulsating stars

Recent theoretical studies of pulsating stars, mainly Cepheids and RR Lyrae, are based on the following computational approaches:

- **Linear non adiabatic hydrodynamic models.** These include the non adiabatic effects related to the driving mechanisms of pulsation but rely on a small-oscillation approximation with the consequent linearization of the hydrodynamic equations describing the pulsation phenomenon. Only the periods and pulsation mode growth rates, the latter determining the instability strip blue edges can be estimated, not the pulsation amplitudes or red edges (unless some linear approximation for convection is included). Nevertheless, interesting results have been obtained in the literature from this approach (see e.g. Anderson et al., 2016; Kovacs & Karamiqucham, 2021; Netzel & Smolec, 2022, and references therein).

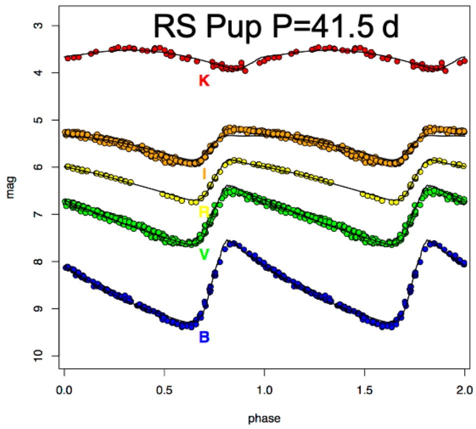

Fig. 1. An example of model fitting of a multifiter light curve for a Galactic Cepheid in the Gaia database. The coloured symbols are the multi-band data from the literature, while the solid lines are the corresponding best-fit model light curves (see Gaia Collaboration et al., 2017, for details)

- **Non linear convective 1D hydrodynamic models.** In this case, the hydrodynamic equations describing the pulsation phenomenon are not linearized so that not only the periods and the instability strip edges can be estimated but also the pulsation amplitudes (full amplitude variation of all the relevant quantities along the pulsation cycle). Several authors developed nonlinear convective pulsation models of Cepheids and RR Lyrae (see e.g. Gehmeyr, 1992; Bono & Stellingwerf, 1994; Kolláth et al., 2002; Bono et al., 1999; Szabó et al., 2004; Smolec & Moskalik, 2008; Paxton et al., 2019, and references in), thus predicting the the location of the boundaries of the instability strips for the first three radial modes as well as the variation of all relevant quantities along the pulsation cycles, including the accurate morphology of light and radial velocity curves and their dependence on the pulsation mode, stellar mass and chemical composition (see e.g. Marconi et al., 2005; Paxton et al., 2019). Moreover, mean magnitudes and colors can be derived from the predicted bolometric light curves converted into a variety of photometric filters. Such (typically intensity-averaged) mean values can be used to build multi-band PL, PLC and Period-Wesenheit (PW) relations (see e.g. Bono et al., 2010; Di Criscienzo et al., 2013).

– **Multi-dimensional hydrodynamic simulations**. In this approach, the 1D assumption is released and the pulsation-convection coupling is treated in 2 or 3 dimensions (see e.g. Mundprecht et al., 2013, 2015). These simulations offer a more realistic treatment of the mechanisms by which the convective motions interact with the dynamical evolution of the stellar envelope, thus providing crucial guidelines for developing descriptions of convection to be applied also in 1D modelling (see also Geroux & Deupree, 2015; Deupree, 2021, and references therein).

3. Recent results for Classical Cepheids from nonlinear convective 1D pulsation models

3.1. The model fitting technique

One of the main advantages of nonlinearity is the possibility to predict the variation of all relevant quantities along the pulsation cycle. This offers the unique opportunity to compare predicted and observed light, radius and radial velocity curves, searching for the best matching model. This model fitting technique, originally applied to a field Magellanic Classical Cepheid by Wood et al. (1997) and to a filed Galactic RR Lyrae pulsator by Bono et al. (2000), allows us to simultaneously constrain the intrinsic stellar properties, that are the stellar properties of the best fit model, and the individual distance of the observed variable star (see also Bono et al., 2002; Keller & Wood, 2002, 2006; Natale et al., 2008; Marconi et al., 2013a,b; Madore, Freedman, & Moak , 2017; Ragosta et al., 2019, and references therein). An example of the application of the model fitting technique is shown in Figure 1 for the multi-filter light curve of the Galactic Cepheid RS Pup. In this case, as outlined in more detail in Gaia Collaboration et al. (2017), the obtained best-fit model is able to nicely reproduce the period, the amplitude and the morphology of the observed light variation in the various filters for an estimate of the individual parallax (0.58±0.03 mas) that was found to be in excellent agreement with Gaia result. Other recent applications to the light and radial velocity curves of Magellanic Cepheids (Madore, Freedman, & Moak , 2017; Ragosta et al., 2019) provided individual estimates of the physical properties for the investigated targets, an indication of rather dispersed Mass-Luminosity (ML) relations and mean Large Magellanic Cloud (LMC) and Small Magellanic Cloud (SMC) distances in agreement with the literature.

3.2. The dependence of predicted observables on the model input parameters

In order to improve the use of pulsating stars as distance (and age) indicators and as stellar population tracers, a crucial step is the investigation of the dependence of model predictions on physical and numerical ingredients. This approach is followed by several authors, even in the most recent literature. De Somma et al. (2020, 2022) computed nonlinear convective pulsation models of Classical Cepheids for various assumptions about the efficiency of super-adiabatic convection, the ML relation, and the chemical composition. For example, Figure 2 shows the effect of the input metal abundance on the predicted instability strip boundaries, as re-adapted from De Somma et al. (2022).

Similar studies have been performed by other authors, such as Paxton et al. (2019), who also investigate the effect of other input parameters used in the convective pulsation code, e.g. artificial and eddy viscosity, but also radiative losses, turbulent flux and turbulent pressure. Moreover, in Bhardwaj et al. (2017) the dependence of multi-color theoretical light curves on model physical parameter, along with comparison with observations, was presented.

The different model assumptions can also be tested through the comparison with observations, not only directly in the color-magnitude plane or for the light and radial velocity am-

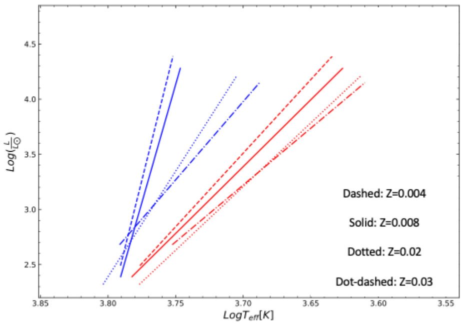

Fig. 2. The metallicity dependence of predicted Fundamental mode instability strip boundaries, as re-adapted from De Somma et al. (2022).

plitudes and morphology but also by comparing the predicted distances obtained from theoretical PL, PLC, and PW relations with measured values. For example, in De Somma et al. (2022) the theoretical metal-dependent PW relations, for different filter combinations and assumptions concerning the ML relation and the efficiency of super-adiabatic convection, have been applied to the sample of Galactic Cepheids with Gaia Data Release 3 parallaxes. The inferred theoretical parallaxes have then been compared with Gaia values, showing a general agreement with the parallax zero-point offset estimated by Riess et al. (2021). Figure 3 displays the difference between predicted and Gaia Data Release 3 parallaxes for a sample of Galactic Cepheids as a function of Gaia parallaxes, assuming canonical (A) and brighter (B) models with standard (left) and enhanced (right) convective efficiency. This kind of comparison allows us on one side to quantify the impact of the selected model assumptions, on the other to constrain these assumptions by relying on the observational behaviours. Inspection of the figure suggests that models including a canonical ML relation but an enhanced efficiency of super-adiabatic convection provide the best agreement with Riess et al. estimate of the Gaia zero-point offset, once Lindegren et al. correction (Lindegren et al., 2021) has been applied to the measured parallaxes (see Riess et al., 2021; De Somma et al., 2022, for details).

4. Recent results for RR Lyrae from nonlinear convective 1D pulsation models

Several efforts have been performed in the literature to model the pulsation properties of RR Lyrae stars, building a theoretical scenario for Zero-Age-Horizontal-Branch (ZAHB) and evolved central helium burning low mass (from ~ 0.5 to $\sim 0.8\ M_{\odot}$ depending on chemical composition) pulsating stars (see e.g. Marconi et al., 2015, and references therein). From this kind of nonlinear convective pulsation models, the variation of luminosity, radius, temperature, and gravity along the pulsation cycle and the topology of the instability strip can be predicted for both fundamental and first overtone modes.

4.1. The modeling of multi-filter light curves and the model fitting technique

The obtained bolometric light curves can be converted into the various photometric bands (see e.g. Di Criscienzo et al., 2004; Marconi et al., 2021, 2022, and references therein) to predict the light curve morphology as well as the mean magnitudes and colors to be used in the construction of PL, PLC and PW relations (see below). Moreover, the model fitting technique can be used to derive the intrinsic stellar properties and the individual distance of the observed RR Lyrae, provided that well-sampled light curves are available (see e.g. Marconi & Clementini, 2005; Marconi & Degl'Innocenti, 2007; Di Criscienzo et al., 2011, and references therein). Recent examples have also been provided by Paxton et al. (2019) for RR Lyrae pulsators in the OGLE database.

4.2. The period-amplitude diagrams

The modeling of multi-band light curves also allows us to predict the pulsation amplitude behaviour as a function of the period (Bailey diagram) for various wavelengths and chemical abundances.

Figure 4 shows the predicted Bailey diagram in the Gaia filters for RR Lyrae mod-

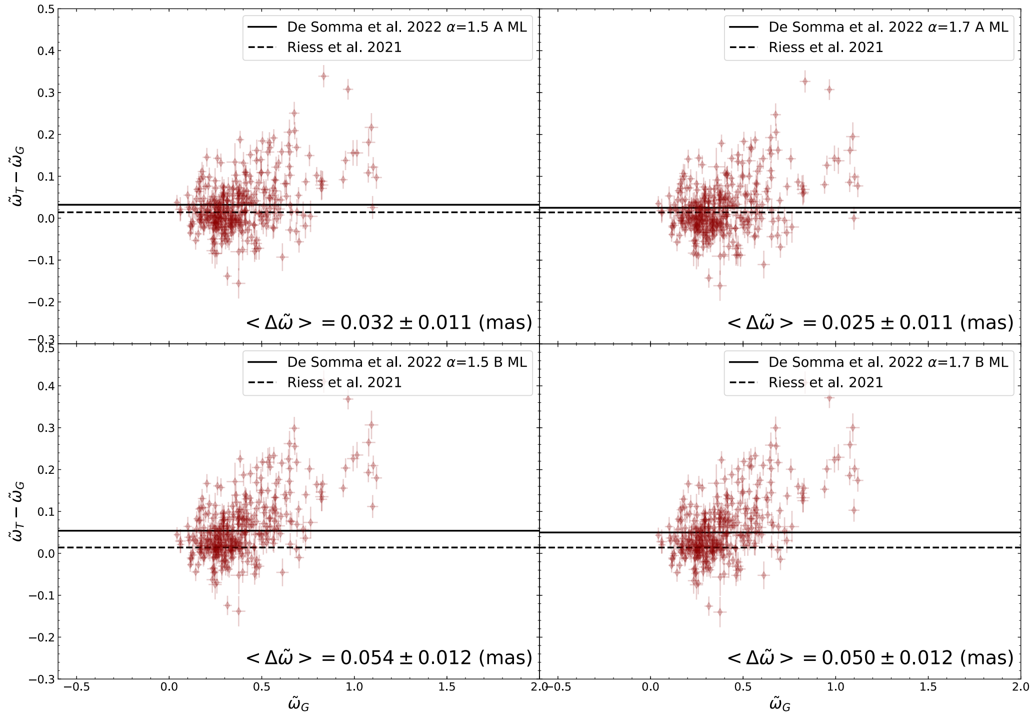

Fig. 3. Difference between predicted and Gaia Data Release 3 parallax for a sample of Galactic Cepheids as a function of Gaia parallaxes, assuming canonical (A) models with standard (α = 1.5, left upper panel) and enhanced (α = 1.7, right upper panel) convective efficiency and brighter models at fixed mass (B) again with standard (α = 1.5, left bottom panel) and enhanced (α = 1.7, right bottom panel) convective efficiency.

els at different metallicity as adapted from Marconi et al. (2021) over-imposed to recent results based on Gaia Data Release 3 (plot adapted from Clementini et al., 2022). We notice that shorter periods and smaller amplitudes are predicted as the metal content increases. Interestingly enough this trend with metallicity is confirmed by Gaia data (colour-scaled dots). Inspection of this figure suggests that brighter models with respect to the ZAHB (dashed lines) predict longer periods than observed, both for Fundamental (F) and First Overtone (FO) modes. The agreement might improve if a higher convective efficiency were assumed in the models as this variation would go in the direction of reducing the predicted amplitude at fixed period (see e.g. Di Criscienzo et al., 2004; Marconi & Clementini, 2005; Marconi & Degl'Innocenti, 2007).

4.3. The color-color plots

An interesting prediction of nonlinear convective pulsation models is the behaviour in color-color planes. For example, in Marconi et al. (2022) the various mean Rubin-LSST colors have been investigated against each other. As a result, the $(g{-}r)$ versus $(u{-}g)$ diagram has been found to be the most dependent on metallicity but with a significantly nonlinear dependence. On the other hand, the $(i-z)$ versus $(r-i)$ plot is predicted to be very little affected by the assumed metal abundance. The most interesting result is the linear relation between $(r-i)$ and $(g-r)$ with a clear dependence on the metallicity of the relation zero-point. This is clearly shown in Figure 5 (see Marconi et al., 2022, for details).

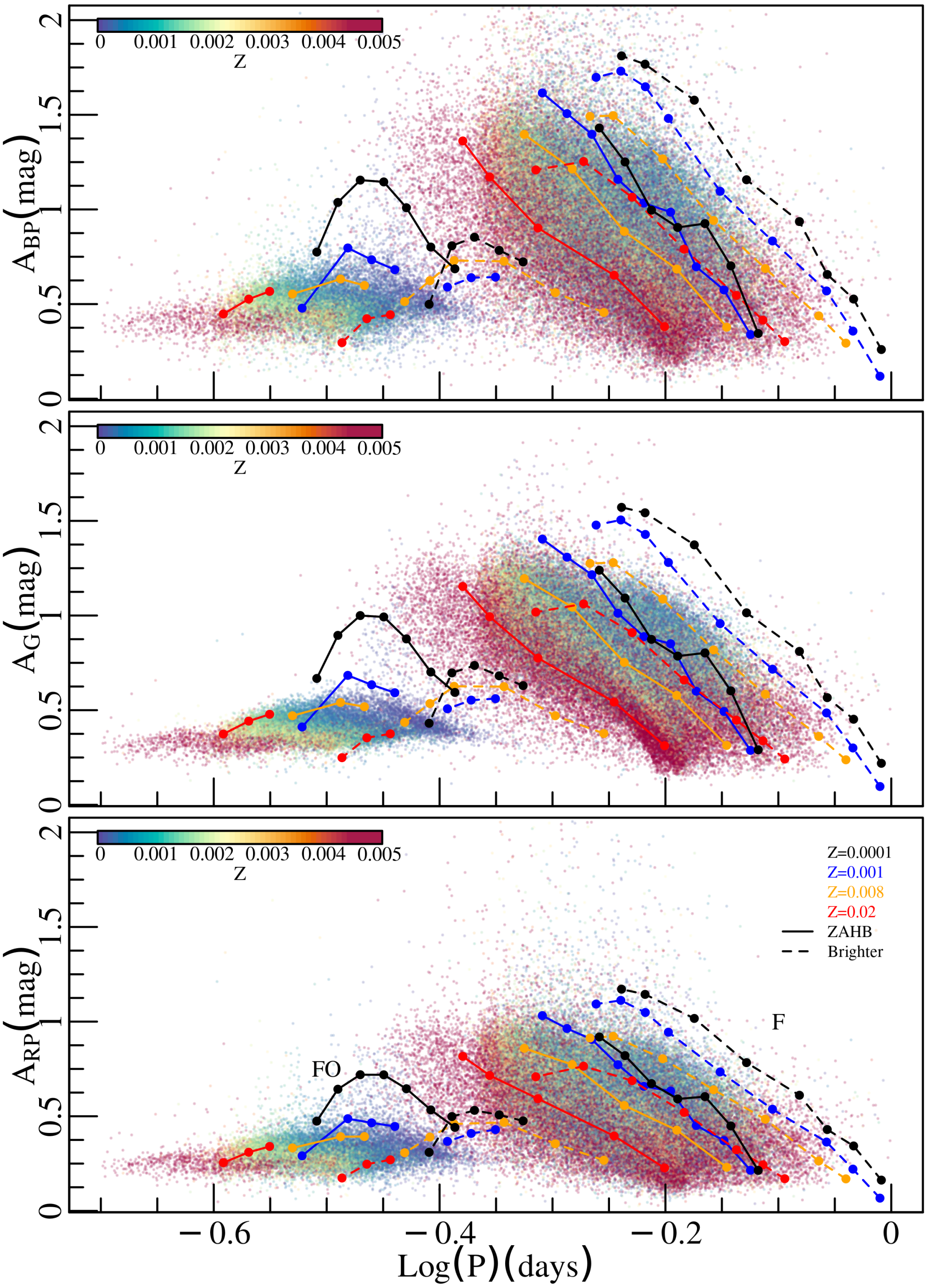

Fig. 4. The predicted Bailey diagram in the Gaia filters for RR Lyrae models at different metallicity (see labelled Z values) both with ZAHB (solid lines) and brighter (dashed lines) as adapted from Marconi et al. (2021) over-imposed to recent results (coloured dots) based on Gaia Data Release 3 (plot adapted from Clementini et al., 2022). For the observations the colour-coded metallicity is obtained through the Φ_{31} − log(P) relation by Nemec et al. (2013).

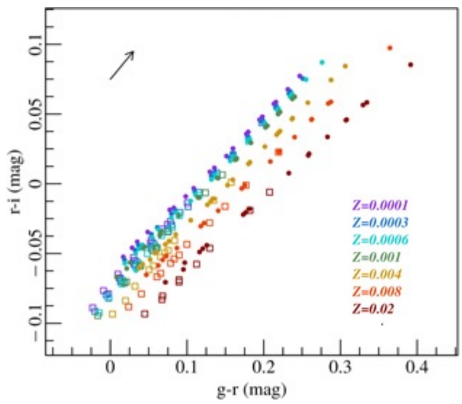

Fig. 5. RR Lyrae model distribution in the $(r-i)$ versus $(g-r)$ relation for the labelled metal abundances. The arrow marks the reddening vector direction.

4.4. the PL and PW relations

RR Lyrae stars are known to obey to Near-Infrared PL relations (see e.g. Catelan et al., 2004, and references therein) and multi-filter PW relations (see e.g. Marconi et al., 2015, and references therein). All these relations are predicted and observed to show a non-negligible metallicity dependence (see e.g. Bono et al., 2003; Marconi et al., 2015, 2021, 2022, for details) with more metal-rich PL relations showing a fainter zero-point but essentially the same slope and PW relations that follow the same trend for most of the filter combinations. This occurrence allowed us to derive theoretical metal-dependent PL (PLZ) and PW (PWZ) relations that are useful theoretical tools to determine individual distances of observed RR Lyrae stars of known metal abundance. Moreover, these relations can be inverted to derive the metallicity distribution of RR Lyrae belonging to the same stellar system of known distance, as recently performed by Braga et al. (2016), Martínez-Vázquez et al. (2016) and Martínez-Vázquez et al. (2021) for RR Lyrae in ω Cen, Sculptor and Eridanus II, respectively.

5. Conclusions

Current pulsation models are able to predict all the relevant properties of classical pulsating stars, in particular Cepheids and RR Lyrae. The multi-filter light curves and the instability strip boundaries show a good agreement with the observations but a non-negligible dependence on several input parameters and physical and numerical assumptions. The model-fitting of multi-filter light and radial velocity curves can constrain the intrinsic stellar parameters and the individual distances. From the predicted periods, mean magnitudes and colors theoretical PL, PLC, PW and Color-Color relations, including a metallicity dependence, can be derived. To improve these predictions, it will be important on one side to test the results against present and future observational data e.g. from Gaia, JWST and Rubin-LSST, on the other to update the physical inputs and adopt more accurate model atmospheres, e.g. moving from static or quasi-static assumptions to the modeling of dynamic atmospheres. Moreover, open problems affected by limitation in the convective treatment, such as the prediction of accurate light curves close to the red edge of the instability strip (see e.g. Marconi & Degl'Innocenti, 2007) or the correct reproduction of double mode pulsators (see e.g. Smolec & Moskalik, 2008, and references therein), should be investigated in further detail. Finally next generation pulsation models should be able to reproduce the debated phenomenon of Blazhko effect among RR Lyrae stars (see e.g. Kolláth, 2021, and references therein).

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