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Memorie della

Rotation curves of galaxies in General Relativity

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Abstract. It has been suggested that the observed flat rotation curves of disk galaxies can be a peculiar effect of General Relativity (GR) rather than evidence for the presence of dark matter (DM) halos in Newtonian gravity. In Ciotti (2022) the problem has been quantitatively addressed by using the well known weak-field, low-velocity gravitomagnetic limit of GR, for realistic exponential baryonic (stellar) disks. As expected, the resulting GR and Newtonian rotation curves are indistinguishable, with GR corrections at all radii of the order of $v^2/c^2 \approx 10^{-6}$. Here we list some astrophysical problems that must be faced if the existence of DM halos is attributed to a misinterpretation of weak field effects of GR.

Key words. Galaxy dark matter halos; Galaxy rotation curves; General relativity

1. Introduction

Following Cooperstock & Tieu (2007, see also Balasin & Grumiller 2008) several papers (among others, see e.g., Crosta et al. 2020; Ludwig 2021; Astesiano & Ruggiero 2022, and references therein), addressed the possibility that the observed flat rotation curves of disk galaxies at large galactocentric distances might be a GR effect characteristic of rotating systems, with no need to invoke the presence of DM halos, as required by Newtonian gravity. In Ciotti (2022) the problem has been studied by using the well known weak-field, low-velocity gravitomagnetic expansion of GR (see, e.g., Mashhoon 2008; Poisson & Will 2014; Ruggiero 2021), by considering, for simplicity, the case of purely stellar exponential disks, with realistic values for the mass and scale-lengths. As expected from the small value of the ratio $v^2/c^2 \approx 10^{-6}$ in galaxies (see point T1 below), the differences between the computed Newtonian rotation curves and the gravitomagnetic curves are everywhere $\approx 10^{-6}$ or less, with the conclusion that in disk galaxies GR requires DM exactly as Newtonian gravity; this conclusion has been recently reinforced by the studies of Lasenby et al. (2023), Glampedakis & Jones (2023), and Costa et al. (2023).

In this contribution I just list some key *astrophysical* problems that should be convincingly addressed by any proposed attempt to replace DM halos in weak-field astronomical systems by GR effects.

2. Theoretical Framework

First, I recall below two general theoretical points:

T1) Observationally, the velocity of stars in disk galaxies is of the order of $\approx 300\,\mathrm{km/s} \approx 0.001\,c$, and proportionally less in lower mass stellar systems (some of them requiring in

Newtonian gravity even larger values of DM-to-baryon ratios than disk galaxies, see next point A4). Classically, for these systems we expect GR corrections to Newtonian predictions of the order of $(v/c)^2 = 10^{-6}$, while in the proposed scenario such corrections should be strong enough to produce a flat rotation curve instead of a Keplerian decline at large radii, a correction several times larger than the Newtonian values! Therefore, the claim that GR in the very weak field, low-velocity limit of real galaxies can produce dynamical effects *dominating by almost an order of magnitude* with respect to Newtonian gravity, is fully surprising, to the point that the consequences on the existence of DM would be of secondary importance. If confirmed, such enormous, unexpected effect would represent a true revolution of our understanding of GR; thus the *physical origin* of such effect should be made as transparent as possible, with the aid of simple models and a mathematical analysis possibly restricted to the essential.

T2) At present, for an astronomer reading the literature, it is disconcerting that no consensus seems to emerge about *what* GR mechanism is actually responsible for the claimed result, with a wide spectrum of suggestions ranging from gravitomagnetism, to GR delicate effects of boundary conditions/vacuum solutions, to retarded potential effects due to unsteady accretion of gas on galaxies (Yahalom 2021), to gravitomagnetic dipole effects produced by pairs of rotating black holes (Govaerts 2023).

3. Observational Framework

From the astrophysical point of view, GR solutions should convincingly address the following points:

A1) First, a warning. Purely baryonic exponential disks (the visible component of disk galaxies) in Newtonian gravity produce reasonably flat rotation curves in the radial range from 1 to 3 scale-lengths: a flat rotation curve inside this region (containing more than 80% of the total visible mass!) *does not necessarily* require the presence of a DM halo for its explanation (see, e.g., Kalnajs 1983; Kent 1986). DM is only *required* by rotation curves in the regions probed by the rotating HI gas, well beyond the edge of the bright optical part of the galaxy (van Albada et al. 1985; van Albada & Sancisi 1986; see also Chapters 20 and 9 in Bertin 2014 and 2022, respectively). Therefore, GR solutions producing a flat rotation curve for the *stellar disk* are not alternatives to DM, but are just in accordance with the common expectation from the Newtonian limit of GR.

A2) If DM is mimicked by GR effects related to the rotation of the baryonic component of disk galaxies, why DM halos are found to be required also in systems with *very low* rotational support, such as Clusters of Galaxies and Elliptical Galaxies (e.g., see Cappellari 2016, Chapter 10 in Bertin 2022, and references therein)?

A3) How is the theoretical point (T1) reconciled with the fact that for systems with gravitational strong and weak lensing (a weak-field limit of GR), DM halos *are* inferred, in remarkable agreement with Newtonian predictions based on stellar dynamics and/or hydrostatic equilibrium of hot, X-ray emitting gaseous halos (see, e.g., Treu 2010, Chapters 7 and 8 in Kim & Pellegrini 2012, Chapter 6 in Meneghetti 2021)? We notice that recently it has been shown that a GR disk model used to explore the impact of GR on the rotation curves of disk galaxies, would also produce enormously large and unobserved lensing deflections (Galoppo et al. 2022).

A4) Dwarf Spheroidal Galaxies (dSph) are low-mass and very gas poor systems, of spheroidal shape, but they appear to have very high DM-to-baryon ratio, even higher than disk galaxies (see, e.g., Mateo 1998; Battaglia & Nipoti 2022). An even more dramatic case of low-mass systems where a significant amount of DM seems to be required is the recently discovered class of Ultra-Faint Dwarf Galaxies (UFDs, e.g. see Belokurov 2007, Simon 2019 and references therein): why GR effects in these systems (with v/c ratios much lower than in normal disk galaxies) should be proportionally more important? And, strictly related:

A5) In Globular Clusters, some of them in the same velocity dispersion/mass range of dSph/UFDs (e.g. van den Bergh 2008), DM *is not* required[1]: why GR should behave so differently in stellar systems of similar mass (see also Fig. 14 in Forbes et al. 2008, Fig. 4 in Wolf et al. 2010, and Fig. 11 in Lelli et al. 2018 for other examples of DM-to-baryon large scatter in stellar systems of similar mass)?

A6) Cosmological simulations with DM and initial conditions derived from the observed CMB fluctuation spectrum appear to be *highly* successful in reproducing the growth of the large scale structure of the Universe. Attempts should be made to obtain comparable results from GR in the absence of DM, similarly to what is currently investigated in MOND, a very well studied alternative to DM on galactic scales. Without entering in the debate about the viability of MOND as an alternative to DM (not the subject of this communication), we notice that several of the points above have been in fact addressed quite succesfully and consistently in the MOND framework (Milgrom 1983, Bekenstein & Milgrom 1984, see also Sanders 2014, Merritt 2020, and references therein), along a line that should be pursued also for the case of GR in absence of DM.

A7) As well known, DM halos in disk galaxies are required to keep the disks stable (Ostriker & Peebles, 1973). Can this stabilizing effect be replaced by GR?

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[1] As it seems for the family of the slightly more massive Ultra Compact Dwarf Galaxies (see. e.g., Mahani et al. 2021).

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