Mem. S.A.It. Vol. 83, 1033

Memorie della

Astrometric cosmology

M. G. Lattanzi

Istituto Nazionale di Astrofisica – Osservatorio Astronomico di Torino, Via Osservatorio 20, I-10025 Pino Torinese, Italy e-mail: lattanzi@oato.inaf.it

Abstract. The accurate measurement of the motions of stars in our Galaxy can provide access to the cosmological signatures in the disk and halo, while astrometric experiments from within our Solar System can uniquely probe possible deviations from General Relativity. This article will introduce to the fact that astrometry has the potential, thanks also to impressive technological advancements, to become a key player in the field of local cosmology. For example, accurate absolute kinematics at the scale of the Milky Way can, for the first time in situ, account for the predictions made by the cold dark matter model for the Galactic halo, and eventually map out the distribution of dark matter, or other formation mechanisms, required to explain the signatures recently identified in the old component of the thick disk. Final notes dwell on to what extent Gaia can fulfill the expectations of astrometric cosmology and on what must instead be left to future, specifically designed, astrometric experiments.

1. Introduction

With the Gaia mission approaching launch, the idea of organizing a workshop dedicated to the implications of micro-arcsecond (μas) astrometry in QSO astronomy and Fundamental Physics finally turned into reality. I was having the irresistible opportunity to put forth my concept of "Astrometric Cosmology". This is the ancient observational science of measuring angles among sources on the celestial sphere brought by technology extraordinary advancements to new accuracy levels. These are able to contribute, in some cases uniquely, to modern Cosmology by confronting its detailed zero-redshift predictions to the observed complexities in the stellar phase space of our Galaxy.

Send offprint requests to: M. G. Lattanzi

2. The new astrometry

There have been several definitions of "new" (or "modern") astrometry over the years in anticipation of, or right after, the release of the data from the Hipparcos mission in 1997 (ESA 1997), the event that brought this ancient observational branch of Fundamental Astronomy to a new life.

Kovalevsky in his seminal essay *Prospects for space stellar astrometry* (Kovalevsky 1984) used the size of the accessible field-of-view (FOV) to classify astrometry [1] thus emphasizing the critical role of technology for providing an ancient science with a prime spot into next century astrophysics: if the prospects of narrow field astrometry could still be granted from the ground (with space as the obvious, al-

[1] From the few arc-seconds of narrow-field astrometry to the 4π of *global*, all-direction, astrometry.

though expensive, alternative) by utilizing interferometry and adaptive optics against atmospheric limitations, going into space appeared as the only option for the future of wide field and global astrometry even at visible wavelengths.
Here, I will adopt a purely operational definition for "new astrometry", so that its different forms discussed in the following sections will appear as logical transitions. Specifically, the following five "key words" are all it is needed for my definition of new astrometry: *all-sky*, *faint magnitude*, *completeness*, *wavelength coverage*, and *accuracy*. These key words synthesize what is necessary today to make progress in wide-field or, better, *global astrometry*: the science of the materialization of the Reference Frame (RF), as I like to call it. Indeed, the definition and realization of the RF, through the implementation of the International Coordinate Reference System[2] (ICRS), is probably one of the most far reaching tasks of fundamental astronomy in the 21st century.

Progress in catalog astronomy[3] has two distinct but equally important sides: one is operational, the other scientific. Common to both is the realization of the RF across the electromagnetic spectrum (*wavelength coverage*) and its extension, or densification, to *faint magnitudes* through a *complete* census of all the sources to a specified magnitude limit. Also, in order of increasing accuracy, i.e. the physics of gravitation utilized, the RF would be called inertial, absolute, non-rotating, or, simply, local (relative to the observer).

Deep space navigation and full (optimal) exploitation of the largest ground-based and space-borne observatories are operational examples that cannot do without faint and complete astronomical catalogs. This is the case of the 8-m class telescopes at facilities like ESO VLT and the Gemini Observatory, whose optimal operations require the acquisition of suitable reference sources (both in magnitude and color) for top performances of their active and adaptive optic systems; or like the Chinese LAMOST (Xiang-Qun Cui et al 2012), which needs to feed and real time operate its 4000-fiber-fed robotic facility.
As for orbiting observatories, operations of the Hubble Space Telescope (HST), including observing proposals preparation, require reliable omnidirectional blind pointing capabilities to minimize orbital maneuvers (especially real-time ones) thus maximizing efficiency (i.e., the fraction of orbital time spent on science exposures) and cost effectiveness. Moreover, the most sensible instrumentation aboard requires protection from stars as faint as $V = 18{-}20$ in a procedure that in the Hubble operations jargon is called "bright objects alert"[4]. This in turn requires 4π completeness of the supporting catalog.
And supporting HST operations was the main driver for the realization of the Second Guide Star Catalog (GSC2): released to the community in 2006 and published in the Summer of 2008 (Lasker, Lattanzi, McLean et al. 2008), the GSC2 lists approximately 1 billion objects, i.e. unique entries, and is complete to the red magnitude $R = 20$. GSC2 is one of the best products ground based catalog astrometry has to offer (see also Monet et al. 2003) possessing all of the traits of modernity introduced above except for accuracy that is limited by the ground-based material from which the catalog was made. Its *scientific* relevance resides in being both the most detailed multi-color view of the Milky Way at optical wavelengths to date, and the faintest and densest materialization at these wavelengths of the ICRF, whose primary realization has been dominated by radio astronomy.

Global astrometry is also the science of measuring absolute stellar distances. Its potential for astrophysics comes from the ability to contribute to the direct, i.e. *model independent*, calibration of radiant and gravitational energy, the two forms of energy that dominate in the

[2] See `www.iers.org/IERS/EN/Science/ICRS/ICRS.html`

[3] This is the name often utilized as a synonym for global astronomy.

[4] Stellar objects in that magnitude range are potentially dangerous sources, as way too "bright" in long exposures imaging of extremely deep fields.

Universe. However, stars are far away and their angular motions are small as they decrease with the inverse of distance itself. Earth's atmosphere has then always posed a serious limitation to increasingly accurate (absolute) distances; and if astrometry was to move away from the immediate solar neighborhood, it was essential to go into space, irrespective of wavelength, therefore requiring tremendous technological challenges.

The success of the ESA mission Hipparcos demonstrated that technological advancements in the last two decades of the 20th century matured to a level sufficient for a first giant step forward. Space-borne global astronomy had finally come of age through the realization of a *self-calibrating* dual-FOV scanning satellite (ESA 1997).

The numbers delivered by the Hipparcos satellite remain unmatched even today, three lustres after the publication of the catalog: 118,218 positions, annual proper motions, and parallaxes down to $V = 12.4$[5], with a median precision of better than 1 *mas*. This is 100 times in number and not less than a 10–fold increase in precision to what produced from the ground since the beginning of modern astrometry in 1838, when the German astronomer Fredrich Bessel measured the first trigonometric parallax establishing the distance to the star 61 Cygni. With Hipparcos, astrometry had finally acquired the capability of producing large numbers of accurate parallaxes throughout the sky. And it was thanks to such an accuracy of genuinely absolute distances that space became undoubtedly the new frontier for global astrometry, which, in turn, established itself as indispensable for solving the open problems in stellar and galactic astrophysics.

2.1. Astrophysical astrometry

What is modern astrometry to do? Its job is to measure, for individual stars and *independently* from models, the following fundamental quantities with increasing accuracy and for the largest samples possible:

1. distance (absolute parallax p);
2. angular position (coordinates);
3. velocity (proper motion μ)[6];
4. mass;
5. photospheric (angular) size ϕ.

This is the realm of *astrophysical astrometry*.

Except for *photospheric sizes*, which are the objective of sub–mas (kilometric) optical interferometry and can be achieved also from the ground, thanks to atmospheric coherence (isoplanatic patches) over the small angles involved, the other quantities must all rely on space astrometry for best results.

The importance of *distance* is obvious as it provides absolute calibration of radiant energy critical for understanding stellar interiors and atmospheric models; also, together with interferometric (photospheric) diameters, distance can constrain the physics governing the unstable phases of intrinsically variable stars, like, e.g., pulsating stars. The challenge is to reach sufficiently large volumes of the Galaxy encompassing samples representative of the complexity of the HR diagram. If distances accurate to 10% are deemed adequate, reaching the kpc scale implies parallaxes to better than 100 μas.

Binary systems can provide access to *masses* across stellar types and evolutionary stages; however, for the mass to be known to better than 3%, i.e. what is required to discriminate among the different stellar models, their distance must be known with a relative accuracy three times better. Therefore, reaching the kpc scale requires, this time, parallaxes ten times better to 10 μas, way beyond what achieved with Hipparcos.

Accurate *coordinates*, or angular position, fix directions on the celestial sphere thus physically materializing the RF. But why is RF materialization important in astrophysics? The answer is that it allows accurate alignment of point-like or resolved emissions at different wavelengths through RF registration, as opposed to "blind" (non-physical) overlap of en-

[5] All–sky completeness was limited to ~ 9 *mag*.

[6] Proper motion provides, with distance, the tangential component of stellar motion; the radial component needed for a full reconstruction of spatial velocity is assumed to be complemented by spectroscopy.

ergy peaks. This is the way for model independent characterization of high energy galactic and extra-galactic phenomena, which are "bright" emitters at, say, radio, X or gamma frequencies but often quite faint at optical wavelengths.
This is the case of the nearby (250 pc away) radio quiet 237–millisec pulsar Geminga. This INS[7] pulsar was optically identified on deep HST images at V=26 mag; this allowed the accurate registration, to 5 mas, of a historical series of X and gamma ray pulses after a 10-mag, 5-step transfer of the Hipparcos (ICRF) RF onto the same HST images (Caraveo, Lattanzi et al. 1998). Theory predicts that pulsar rotation slows down as $d\Omega/dt = -k\,\Omega^n$,where $\Omega = 2\pi/P$ is the rotational frequency (P the corresponding period), k is a positive constant, and n is the *breaking index* that describes how the pulsar spins down. Simple modelling of pulsar radiation predicts $n = 3$; therefore, the possibility of determining n independently from theory provides direct insight into the physics of pulsars. The registration of the pulse timing through accurate astrometry allows the determination of the first and second derivatives of the spin frequency (or period) and therefore of the breaking index via the relation $n = (\Omega\,\ddot{\Omega})/\dot{\Omega}^2$, and of the pulsar characteristic age as $\tau = \frac{1}{1-n}\,\frac{\Omega}{\dot{\Omega}}$, or its analogue in terms of the spin period $\tau = \frac{1}{n-1}\,\frac{P}{\dot{P}}$.

In the radio domain, VLBI and VLA have since a few decades reached accuracies and resolutions exceeding the milli–second of arc. And RF registration between radio maps and optical images of the same objects, or, better, of the same regions of the sky, has been instrumental in testing structural and energy models of active extragalactic objets. In 1997, Lattanzi, Capetti & Macchetto extended the point–like source technique utilized for the accurate absolute positioning of Geminga to Seyfert 2 galaxies. The bright Hipparcos-based ICRS frame was first transferred to faint HST images of the class prototype NGC 1068 taken in visible light (continuum and [OIII] narrow line emissions) through specifically procured ground–based material for extending, and therefore densify, the primary RF to fainter magnitudes. Then, the registered space–borne images were overlaid onto MERLIN 6-cm maps of the same radio loud galaxy to explore the *spatial relationship* of the two types of emission. The results were quite telling: significant anti–correlations existed in the angular energy distributions at the different frequencies, bringing solid evidence to the validity of the Unified Model theory for Seyfert galaxies (Capetti et al. 1997).

[7] Isolated Neutron Star

Distances to the stars are not only the key to absolute radiant energy but also to the gravitation energy content of the Milky Way (MW). For, distances fix, along with space *velocities*, the initial (boundary) conditions for MW dynamics. This time, reaching the kilo–parsec scale requires that not only parallax, but also proper motion must be known to better than 7% for spatial velocities, V_t, accurate to 10%, as per the error propagation formula $(\sigma_{V_t}/V_t) = \sqrt{(\sigma_\mu/\mu)^2 + (\sigma_p/p)^2}$ (the σ's indicating standard errors). Measuring individual stars, i.e. mapping the MW phase space *in situ*, provides accurate probing of its most prominent constituents (bulge, disk, halo) required to confront with the most sophisticated, cosmology driven, high resolution models of MW dynamics. I will briefly go back to this point in the following sections.

2.2. Relativistic astrometry: the Gaia era

As mentioned earlier, thanks to Hipparcos, space astrometry demonstrated its technological maturity. With the new century approaching and technology advancing fast, precision astrometry was ready for a second giant step. However, one gap needed to be closed: extending accuracy to faint magnitudes. This meant realizing in space a large scale catalog (similar to, e.g., the GSC2) following the precepts of the Hipparcos mission: the Gaia mission concept, visually shown in Fig. 1, was finally born (see Perryman et al. 2001, and references therein)!

The Gaia portal at `www.rssd.esa.int/index.php?project=GAIA\&page=index` provides a detailed and constantly updated

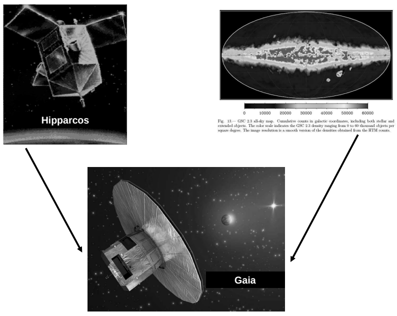

Fig. 1. The Gaia paradigm: build an all-sky large and deep catalog, pretty much like the latest generation of ground-based surveys (e.g., the GSC2), by going into space (for maximum accuracy) following a mission profile similar to that proved with Hipparcos.

account of the status of the mission, which will be launched by ESA not earlier than October 2013. Of relevance here is the expected end-of-life astrometric performance shown in Tab. 1 and released by ESA late last year following the positive conclusion of the payload critical design review. Comparing the listed figures to the requirements discussed in sec. 2.1, these appear largely satisfied depending on target magnitude and color. Astronomers wishing to assess the impact of the Gaia mission on their own research fields should start from Table 1. This is the result of averaging over the five-year mission and across the latitude dependent number–of–observations distribution. Therefore, science programs investing variable phenomena, i.e. requiring epoch observations, or concentrated along particular lines of sight should refer to the detailed information available at the Gaia www portal (section on Gaia's error budget) before establishing feasibility. There, they can also find more information on the actual error statistics, with indication on the expected presence, and level, of possible correlations and of residual systematic errors.

Because of the choice to operate Gaia in the visual domain, interstellar extinction will cause difficulties in penetrating the kilo–parsec scales at all longitudes in the MW disk, which requires going to redder wavelengths. In this context, the effort by the Japanese Space Agency (JAXA), through the JASMINE program (Gouda 2011), is of particular value: after a demonstrator, *nano-JASMINE*, to be launched in November 2013, two more satellites will follow, with the medium-class mission JASMINE, anticipated to be launched in 2020, operating in the K_w band (1.5 - 2.5 μm) and targeting 10 μas at $K_w = 11$.
And, hopefully, NASA will rethink its strategy by going back to pursue missions like JMAP and the sub-μas capabilities of the SIM-light project (Unwin et al. 2008).

At those accuracies light does not propagate in straight lines and time does not beat the same everywhere: photons follow geodesics and physical time is only that of the observer: welcome to the land where Einstein's General Relativity rules!
Astrometry becomes fully relativistic even in weak gravitational fields, where the corresponding weakly relativistic metric tensor, $g_{\alpha\beta}$, can be given in terms of the perturbation, $h_{\alpha\beta}$ ($|h_{\alpha\beta}| \ll 1$), to the flat Minkowskian metric, $\eta_{\alpha\beta}$, as $g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta} + O(h^2)$, i.e., the background geometry is sufficiently small that non–linear terms can be neglected. In a gravitationally bound system, the virial theorem assures that all forms of energy density within the system cannot exceed the maximum amount of its gravitational potential U; therefore, in the presence of a weak field it must be $|h_{\alpha\beta}| \lesssim U/c^2 \sim \mathrm{v}^2/c^2 \ll 1$, where $U = GM/D$, v is the characteristic velocity within the bound system, and D represents the system linear size. That this situation applies to our Solar System (SS) can be easily verified by recalling that v $\sim$ 30 km/sec is its typical internal velocity, and $D \sim 70$ au its characteristic extension.

This conference dedicated a full day to the subject of relativistic reference frames and the description of space–time fabric (see the contributions by Klioner, Kopeikin, and Crosta in this volume). In the context of the Gaia mission, two different formulations of relativistic light propagation have been developed to model astrometric observations

Table 1. Gaia End-of-life averaged astrometric performance; Req.= Required and Perf. = performance, as estimated after the payload acceptance review (courtesy of Jos de Brüijne).

μas	B1 V Req.	Perf.	G2 V Req.	Perf.	M6 V Req.	Perf.
V < 10 mag	< 7	8.4	< 7	8.6	< 7	10.6
V = 15 mag	< 25	26.3	< 24	24.4	< 12	9.4
V = 20 mag	< 300	328.7	< 300	292.8	< 100	97.7

of distant sources by a SS observer: the GREM formulation, the well known coordinate based approach described by Klioner (2003 and references therein), and the RAMOD model, the approach fully compliant with the precepts of local measurement in a relativistic setting discussed by Crosta (2011, and references therein). Their theoretical equivalence, to the 1-μas accuracy level suitable for Gaia, has been recently demonstrated (Crosta and Vecchiato 2010, Crosta 2011)[8] and will be exploited, in a process called, in the Gaia jargon, Astrometric Verification (or AVU; see Lattanzi et al. 2006), by comparing the results of two fully independent astrometric reconstructions of the celestial sphere to assess all-sky scientific reliability on positions, including parallax, and proper motions.
However, what I wish to emphasize here is that these recent investigations in light propagation and direction measurement have finally provided what I like to call the *basic equation of relativistic astrometry*:

$$\bar{l}^{(i)} = n^{(i)}(1 - \frac{h_{00}}{2}) + O(\frac{\mathrm{v}^4}{c^4}) \ , \qquad (1)$$

where $h_{00} = 2U/c^2$, which is Eq. (25) in Crosta and Vecchiato (2010). This equation provides the components of the spatial light direction $\bar{l}^{(i)}$ in terms of its Euclidean counterparts $n^{(i)}$ (used in classical astrometry) at the observer (satellite) location in the gravitational field of the solar system (h_{00}). $\bar{l}$ is the direction for the local barycentric observer, i.e., for the observer whose frame is at rest relative to, and its axes have the same orientation as, that at the SS barycenter. With reference to Fig. 2, the actual (observed) direction in GR, i.e. the analogue of vector S, the aberrated direction seen at the time of observation by a classically modelled observer on the moving satellite (SRS)[9], is obtained by a simple, Minkowskian, boost transformation (see Crosta and Vecchiato, 2010). The generalization to the full problem, i.e., to the solution of the photon trajectory is given in Crosta (2011).

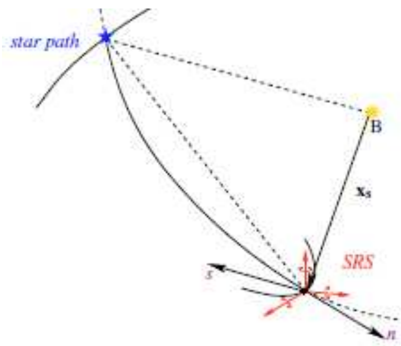

Fig. 2. The vectors representing the light direction in the pM/pN approaches inside the near-zone of the solar system.

[8] This is one of the most important results of the Gaia Data Processing and Analysis Consortium (DPAC) collaboration in the field of relativistic astrometry

[9] At the instant of observation the origin of the local barycentric frame coincides with that of the moving observer (satellite)

3. Astrometry is "local" anyway!

The μas accuracy achievable with space–borne astrometry allows astronomers to reach the kilo–parsec (kpc) scale of the Milky Way, yet this is not enough to probe directly the Mega–parsecs of extragalactic distances (see Tab. 2); this would require angular accuracies in the nano–arcsec regime, beyond today's and, probably, near future technology. Therefore, despite its tremendous improvements over the last very few decades, space astrometry continues to share one key trait with its ground-based traditions: *individual distances or, better, "in situ" investigations can only access the local universe*.

On the other hand, improvements in cosmological models have recently produced quantitative predictions on the present–day consequences of the evolution of the Universe, something that is often referred to as *Local Cosmology* (LC).

These "cosmological consequences" could manifest themselves as characteristic signatures in the main constituents of the MW (halo, disk) or small perturbations in the gravity in action within our SS. As we will see, some of these perturbations are well within the reach of Gaia's astrometry. Therefore, astrometry can, once again, contribute *direct* and *model independent* tests not just of astrophysics, but, this time, of *cosmology*!

4. Cosmology at the local scale

Is it really possible to investigate through "local measurements" on the nature of the Universe? Does LC really exists?

4.1. Fossils in the Milky Way

To find out about signatures at z = 0, one has to turn to predictions from the most sophisticated cosmological simulations, which are built following the adoption of a universe model.

Among the available universe models the *s*tandard (cosmological) model (SCM), or *concordance model*, is the one that has convimcingly explained the tiny (in amplitude) cosmic microwave background (CMB) fluctuations as observed by the WMAP satellite and other CMB experiments (Seife 2003).

Therefore we looked at the predictions of available simulations based on such a model, i.e., a flat universe whose total relative density (Ω =1) is dominated, $\Omega_\Lambda \simeq 0.70$, by the dark energy, or Λ, component, while the part contributed by matter, Ω_m, is mostly made of cold dark (CD) particles (25%), leaving a meager $\simeq$ 5% to ordinary (baryonic) matter (Sawangwit and Shanks 2010).

The paradigm of these Lambda Cold Dark Matter (ΛCDM) models, in keeping with the "merger tree" of Lacey and Cole (1993), is that smaller DM haloes merge at higher redshifts ($z \geq 3$) to form the large structures observed today (z = 0). This could be the way the halo of a massive spiral like the MW formed. The key fact here is that the imprint of these merger events remain as conspicuous "fossil" signatures in the phase space of the MW halo has shown in Fig. 3. The Gaia satellite has been conceived as the ultimate phase-space machine: by measuring distance and velocity of individual stars to sufficient accuracy and within e few kpc from the Sun, the satellite is required to reveal and characterize in-situ the patterns emerging form the halo accretion history shown in Fig. 3, i.e. map the stars belonging to the different "strings", thus accurately testing present-time predictions of the ΛCDM Universe. In fact, the Gaia end-of-life performance table presented in sec. 2.2 tells that a 10% error on distance is still achieved at 10 kpc from the Sun (that is the galactocentric distance from 10 to 20 kpc in Fig. 3); this confirms that Gaia will examine with unprecedented clarity, through its individual measurements of the proper tracers, the fine six-dimensional structure of the inner halo, i.e., within $3 \div 4$ kpc of the Sun location.

The halo is not the only MW structural component with remnants of, or clues to, the formation and early evolution of our Galaxy. Within a distance envelope similar to that of the inner halo, i.e. for distances from the galactic plane in the range $1 \div 3$ kpc, Spagna, Lattanzi et al. (2010) found evidence of a correlation between (galactocentric) circular velocity (V_ϕ) and metallicity ($[Fe/H]$) with an

Table 2. The local Universe. Distances to some of the closest and best known sources outside the MW. The parallax error indicated, set to 10% of the corresponding parallax (p), is the minimum required for astrometric, in situ, access to the scale of the Universe just around our Galaxy. The nano-arcsecond (nas ≡ nano-arcsec, i.e., 10^{-9} arcsec, or $\sim 4 \times 10^{-14}$rad) is required to reach astrometrically the closest quasars

Object	Sky direction	redshift (z)	distance (Mpc)	σ_p (10% p) (μas)	Comment
LMC	Dorado/Mensa	0	~ 0.1	~ 1	
M31	Andromeda	0	~ 0.8	~ 0.1	
M33	Triangulum	0	~ 0.9	0.1	
M81	UMa	~ 0	~ 4	~ 0.03	
M77 (NGC1068)	Cetus	~ 0.01	~ 19	~ 0.01	Seyfert 2 prototype
3C405	Cygnus	0.04	~ 150	≤ 1 nas	Closest quasar.
3C273	Virgo	0.2	~ 749		Optically brightest quasars in the sky (at V≃ 12.9).

estimated gradient of $\partial V_\phi/\partial[Fe/H] \simeq 40 \div 50$ km/sec/dex derived from a sample of about 27,000 stars with metallicity -1 < $[Fe/H]$ < -0.5 dex that was adopted as an unbiased tracer of the thick disk. In a recent paper Curir, Lattanzi et al. (2012) explained, through extensive numerical simulations, that this strong rotation-metallicity relation can be the result of: *(i)* the natural dynamical evolution (associated to heating and radial migration) of a long-lived disk population of main sequence dwarf stars in the gravitational field of a massive MW-like DM halo consistent with a ΛCDM model; and *(ii)* the presence of a cosmologically plausible radial metallicity gradient with *lower* metallicity in the inner regions (i.e., an "inverse", positive, gradient) in the early Galaxy. Indeed, a positive rotation-metallicity correlation , in all similar to that observed by Spagna, Lattanzi et al. (2010), develops rather quickly *without the need of any merging events* and remains stable over several Giga years to an age compatible with that of the observed disk population (~ 10 Gyr).

That an inverse (positive) chemical gradient, whose fossil manifestation shows up in today's thick disk, could exist in the early MW is proved by the findings of Cresci et al. (2010), who studied a sample of distant galaxies at redshift z ≥ 3 and found evidence for an inverse metallicity gradient, possibly produced by the accretion of primordial gas. If we utilize the formula for the Einstein-de Sitter (flat) universe

$$1 + z = \left(\frac{t_0}{t_e}\right)^{2/3} , \qquad (2)$$

(Shu 1982, p.378), then the "emission" time t_e of the photons at z = 3 is $\sim$ 2 Gyr given that the epoch of observation (present time)[10] t_0 is ≈ 10 Gyr; this is consistent with the epoch at which Curir, Lattanzi et al. (2012) injected the positive chemical gradient in their purely N-body simulation.

Although the Spagna, Lattanzi et al. (2010) discovery has been confirmed by several independent studies (see Curir, Lattanzi et al. 2012 for a review), the significant errors that afflict the extant, limited, data on which these studies are based are still the main limitation to a detailed theoretical understanding of the structural and chemical history of the Galaxy's thick disk, although indications are already there that the ΛCDM model seems, at best, to be able to account for only part of the story that made the MW what it is today. This is something Gaia is in a unique position to settle once and for all,

[10] In the same universe $t_0 = (2/3)\times {H_0}^{-1}$, i.e., ~ 10 Gyr for a Hubble constant of H_0 = 70 km/sec/Mpc

as anticipated in Re Fiorentin et al. (2012, and references therein).

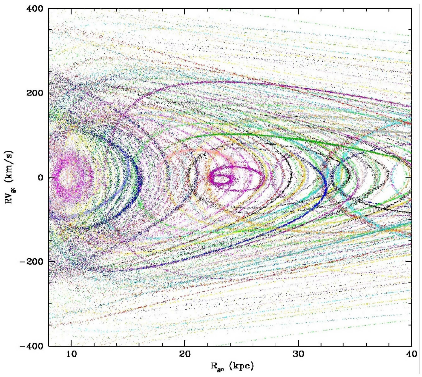

Fig. 3. Milky Way reduced phase space (galactocentric line-of-sight velocity component,RV_{gc}, vs distance from the galactic center, R_{gc}) showing the highly structured fossile features ("streams" or "strings") resulting from a ΛCDM simulation of 100 dwarf galaxies merging with the MW DM halo. The diffrent strings-like structures clearly differentiate the individual merging events.

4.2. Quasar astrometry

Why are quasars mentioned here? It was just concluded that the closest quasar is so far away that even nano-arcsec astrometry could not 'locate' it to the required (better than 10%) accuracy. The answer is in the fact that cosmology is usually associated not with large distances, but rather with *expansion*, i.e., a cosmological velocity. Now, let us follow the suggestion, the "temptation", that the cosmological velocity is not just pure recession (*spherical* expansion) but is more general in nature admitting a transverse component, i.e., "perpendicular" to the traditional line-of-sight component, measured through spectra and used in the Hubble law $V_{rec} = H_0 \times d$, with the *proper* distance d expressed in Mpc when H_0 is in km/sec/Mpc. If such a *rotational* (tangential) component to the cosmic expansion exists, then it would have to do with the appearance of *proper motions* of the distant quasars through the conversion relation $V_{rot} \propto \mu \times d$, with V_{rot} in km/sec, μ the corresponding *cosmic* proper motion, in arcsec/yr, and d usually in pc.

This is the kind of astrometric cosmology mentioned by Eubanks (1991); he reconsidered radio interferometric quasar astrometry for use in cosmology thanks to the fact that, back then, VLBI was already able to pin point quasar sources to z∼ 3 with sub-mas precision in a single observing session.

Therefore, if we admit that recession and rotational components contribute equally to the cosmic expansion, then we can set the scalar relation $V_{rec} \simeq V_{rot}$, that does not depend on distance and provides the means to evaluate the level of cosmological proper motion given the current estimates for H_0. Setting $H_0 = 75$ km/sec/Mpc one has $\mu \simeq H_0/(5 \times 10^6) \simeq 15$ μas/yr, with the effect increasing for larger values of the Hubble constant.

According to Table 1, this number is certainly within the capability of the Gaia mission, once we factor in the large number of quasars that the satellite will observe up to redshift $z < 3$. And the suggestion is there that the previous relation can be inverted for an independent estimate of H_0 from quasar's all-sky astrometry, the only serious limitation being the actual accuracy (as opposed to precision) of the measured proper motions.

Finally, it is worth emphasizing that only the most astrometrically stable quasars could be used in this kind of cosmology work and their existence, i.e. the precise characterization of their astrometric history (extensively discussed at this meeting), might enable another long sought for discovery, that of fossil gravitational waves, echoes of catastrophic (energetic) events reaching us from the deep Universe.

4.3. What if gravity deviates from GR?

If GR is the correct description of gravity, than ordinary matter is just a small portion of what "there is" in the Universe. The concordance model rules, GR is valid everywhere, and a cosmological constant is associated to the exotic components of *dark matter* and *dark energy*, as mentioned in sec. 4.1.

The small ($\sim 10^{-5}$) amplitude fluctuations measured by the WMAP satellite in the CMB

temperature [11] are thought to be the seeds that led to the formation of larger and larger structures and eventually to galaxies through the merger tree paradigm discussed in sec. 4.1 in the context of the ΛCDM model. However, the physical explanation for the origin of such CMB variations seems to elude GR. For, those ripples could be the result of fluctuations in a scalar field (see below) that drove inflation, i.e., the phase of *accelerated* expansion in the evolution of the Universe just before the afterglow light pattern that we observe as CMB today. Therefore, there might have been an epoch in the early Universe when gravity was not following GR. This of course does not directly challenge Einstein's theory supporting SCM, today unquestionably the best in describing the Universe that emerged afterward.
Yet, the existence of large galaxies (possibly the largest) and clusters of galaxies at high redshift (Sawangwit & Shanks 2010), and the apparent lack of, or less than anticipated, merging activity in the formation of the MW halo and thick disk (sec. 4.1), are mounting astrophysical evidence against the ΛCDM model. Also, the same accelerated expansion observed at an epoch ($0.2 \lesssim z \lesssim 0.6$) much more recent than CMB, through Type Ia Supernovae (Riess et al. 1998), has required the introduction of exotic matter terms, including dark energy responsible for repulsive gravity, in order for GR to hold. Might it be, then, that cosmological (CMB) and extragalactic observations are probing the breakdown of GR at large scales?

The SCM new components of the universe, i.e., the new sources of gravity, can be dealt with through the use of the modified field equations Einstein introduced himself, i.e.,

$$G_{\mu\nu} + \Lambda\, g_{\mu\nu} = \frac{8\pi G}{c^4}\, T_{\mu\nu} \ , \qquad (3)$$

where $G_{\mu\nu}$ is the Einstein tensor[12], $g_{\mu\nu}$ is the metric tensor introduced before, Λ the cosmologial constant, and $T_{\mu\nu}$ the stress-energy (or energy-momentum) tensor representing the gravity sources. $T_{\mu\nu}$ can 'absorb' the dark matter component of the SCM universe and, in this same context, the term in Λ is the same (to within a proportionality factor) as an intrinsic energy density of the vacuum; as such, it is commonly moved onto the right-hand side of Eq. 3 becoming part of the field sources. For, if this kind of energy density is positive, the associated negative pressure can drive an accelerated expansion of the universe, as observed.

Now, while waiting to unravel the physical nature of those new constituents dominating (over regular - baryonic - matter) the SCM universe[13], it could well be, using similar arguments, that it is the geometric terms on the left-hand side of Eq. 3 that need modification. Different possibilities for modifications of gravity have been put forth that can contribute both at galactic (e.g., galaxy rotational curves) and beyond galactic (i.e., cosmological) scales, and they all challenge GR and seek validation through the same observational data that have so far corroborated the success of the GR supported SCM.

The subject of modified gravity theories has received quite a lot of attention at this conference(see Bertolami and Capozziello in this volume) in view of the special role that μas astrometric observations can have in testing gravity at the local scale, adding and complementing to the data available, or soon to become available, at the galactic, extragalactic, and cosmological scales.

The *f*(*R*) gravity theories have the potential to realize realistic cosmology (cosmological acceleration), galactic dynamics, etc. without non-baryonic dark matter and dark energy. They emerge from the consideration that there is no fundamental reason for the Ricci

[11] These fluctuations in the microwave sky are small in amplitude but quite conspicuous in angular size reaching the 1°, or twice the size of the full Moon (Sawangwit & Shanks 2010, and references therein)

[12] $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu}$, $R_{\mu\nu}$ is the Ricci tensor and R the Ricci scalar or scalar curvature.

[13] The burden is indeed on the Standard Model of particle physics to break the degeneracy and find the exotic particles (e.g., the WIMPS) in the lab!

scalar to appear in its simplest (linear) form, i.e. $f'(R) = 0$, in the field equations of GR (Capozziello & Faraoni, 2011). This is perhaps what makes $f(R)$ gravity appealing, it goes beyond GR without giving up its fundamental structure. This generalization of the scalar metric R can then be constrained by, e.g., astrometric measurements from within the SS through expressions like

$$\gamma_R^{PPN} - 1 = \frac{-[f''(R)]^2}{f'(R) + 2[f''(R)]^2} \quad , \qquad (4)$$

relating the first and second derivatives of $f(R)$ to the deviation of the PPN parameter γ from its GR (=1) value (Capozziello & Troisi, 2005). Such a deviation can be measured as an extra astrometric component to the relativistic direction of distant objects as seen by a SS observer like the Gaia satellite. Similar deviations are predicted by *scalar-tensor* theories (e.g., Fuji & Maeda, 2003; Ferreira & Starkman 2009, and references therein), another class of alternatives to GR capable of a unified picture (i.e. from the cosmological to the local scale).
Therefore the measurement of the PPN main parameters can be used to test at the very local scale, i.e. within the 'cosmological bubble' of the MW at $z = 0$, these modified gravities, providing local verifications to cosmological constraints. The possibility to execute these tests with Gaia is discussed in the next section.

4.4. Cosmology from within the Solar System

As mentioned earlier, the ripples present in the CMB observed distribution could be the result of fluctuations in a scalar field that drove inflation, and this is the view of the *scalar-tensor* theories (see, e.g., Damour and Nordtvedt, 1993). There is the possibility that this inflation field, which couples with gravity, fades with time. Today (z=0), its residue would manifest itself through very small deviations from Einstein's GR. Astrometric observations can be the means to trace back the presence of this scalar field, through accurate measurements of deflection of the light coming from bright and angularly intrinsically stable sources. The deviation ($\gamma^{PPN} - 1$) predicted in Damour, Piazza & Veneziano (2002) is particularly tiny, in the range $10^{-8} \div 10^{-5}$ (see also Vecchiato et al. 2003, and references therein). The larger value has already been excluded by the results based on radio links to the Cassini spacecraft (Bertotti et al. 2003). Vecchiato, Lattanzi et al. (2003) showed that the Gaia nominal mission could measure γ, i.e. its deviation from unity, to $\sim 10^{-7}$ (1 σ) after 5 years of continuous observations, and using a subset of approximately 10^6 stars chosen as the most astrometrically stable among the millions available in the magnitude range V $\leq$ 12 of the GAIA survey.
Those results, scaled to the predicted performance of the as-built astrometric payload (Table 1), brings the error on γ closer to 10^{-6}. This number implies that GAIA will certainly extend by more than an order of magnitude the chance of probing possible local deviations from GR, yet significantly away from the lower bound of the interval above. Unquestionably then, closing the gap with the local effects predicted by a scalar-tensor gravity cosmology requires the 1 μas accuracy level, therefore establishing the need to go beyond Gaia, possibly toward an astrometric mission fully dedicated to the physics of gravitation.

What if not even the smallest predicted deviations from GR are detected at the local scale? It could be evidence not just of problems with scalar-tensor theories over GR, as GR appears to be inadequate at epochs closer to the Big Bang, but that 'reality' is more complicated and that is why we will not know until gravitation and quantum mechanics are reconciled (brought together). Or, until a completely different view of the physical world (perhaps GR and QM cannot be reconciled!) will come into play.

5. Micro-arcsec precsion and beyond

The previous sections have definitively and quantitatively established the realm of LC; they have also made clear that LC would greatly benefit from actually reaching the 1-μas accuracy level, already exceeding Gaia capabilities in certain circumstances, like the experiments from within the SS.

Cosmology or fundamental physics are not the only reasons for reaching the μas and sub-μas astrometric precision. The detailed characterization (i.e., accurate ephemeris) of rocky and down to Earth-mass planets for atmospheric analysis and, in a non-too-distant future, imaging of those systems more likely to be similar to our own requires 1-μas *single epoch* measurements and possibly better: given that the reflex motion induced by Jupiter on our Sun, when seen by an observer at 10 pc, amounts to 500 μas, the pull due to Earth from the same distance scales as the product of the planet mass ratio (~1/300) times the variation of the orbital sizes (~1/5.2) to 0.3 μas!

It is therefore time we ask ourselves the question: what is a “micro-arc-second”?

Unlike *resolution* in imaging astronomy or interferometry, astrometric accuracy, or astrometric resolution, can be understood as the ability to reconstruct, or maintain, the line-of-sight of a given imaging system to within the desired angular accuracy, so that any changes in direction of the light coming from a distant source is intrinsic to the source and not to the photon collecting (and recording) device, thus available for further astrophysical investigations.

Fig. 4 is an attempt at “materializing” the concept of one-micro-arcsecond astrometric accuracy. It shows the sketch of a 2-m flat mirror, roughly the size of the two Gaia’s primaries, with the vector normal to the surface representing its line-of-sight. A change of ~ 1μas, i.e. ~ 5 pico-radians, in the line-of-sight direction translates into a mirror displacement, actually the quantity that makes sense metrologically, of ~ 5 pico-meters, 100 times smaller than the atomic dimensions!

Is the pico-meter level of *stabilization* actually accessible?

There are two possibilities for lowering instrumental line-of-sight jitter of large payloads (say, a ’cube’ of ≈ 4 m on a side, like in the case of Gaia): passive and active stabilization.

Active stabilization to the pico-meter level was the first to be proven in the laboratory both in Europe, as part of a technology enabling program funded by ESA in the context of the development phase of the Gaia program (Bertinetto & Canuto 2001), and in the US, by the outstanding work of Shao’s JPL team designing and experimenting technology for the different variants of a (sub-) μas space interferometry mission (Shao 2006; see also Unwin, Shao et al. 2008, and references therein).

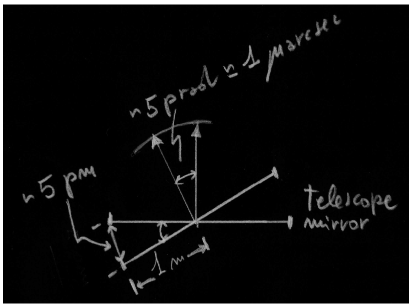

Fig. 4. Visual representation of the μarcsec astrometric accuracy, i.e., of the line-of-sight stabilization/reconstruction, in terms of linear quantities roughly representing the typical size of optical payloads of satellites like Gaia (main mirror size ~ 2 m).

However, active stabilization requires spatialization of sophisticated technology needing real time control logics (like, e.g., servo loops) to be operational for the relatively long time spans usually employed by astrometry (to resolve time dependent quantities, i.e., parallax and space velocities). This considerably rises the risk level, let alone costs, of already challenging programs.

That is why passive means have been preferred so far, like in the case of the Gaia satellite (payload and service module). Nevertheless, passive technology has required its own developments and validations, relying on the properties of new materials (SiC was selected for Gaia) produced and assembled in structurally homogeneous payloads, and enclosed in environments tens of cubic meters in volume to be kept thermally quite everywhere, again passively, to better than 10^{-5} °K, a daunting challenge in itself.

Finally, the two space instrumentation building methods considered for reducing instrumental jitter also differ in their notion of observing strategies and systematic errors control.

Active stabilization schemes emphasize the use of specific on-board technology for continuous instrumental calibration and monitoring, thus endowing astrometry with the capability to reach 1 μas in a *single epoch measurement* (*time resolved μas astrometry*) and, consequently, that to average multiple epoch measurements to well below 1 μas [14].

In a pure passive stabilization strategy, besides the passive thermal control, it is the astrometric payload itself that "does it all" during science operations, being the only hardware designed for high precision measurements; also, instrumental design minimizes systematics. The required error on the program objects is reached only at the end of the on-ground data processing, when all of the elementary exposures are assembled together.
Also, the same high precision measurements on the detected sources (stars or star-like objected) are utilized for assessing and, possibly, modelling the residual systematic biases through what is called, in the case of a spinning satellite akin Gaia, the *closure condition*: as the satellite re-observes the same sky regions after completing one revolution some hours later, any changes in direction over this time interval for the most astrophysically stable sources in the field cannot be due to parallax effects or intrinsic motions, they must be attributed to tiny changes within the instrument. Thus, the fine instrumental behavior, and therefore systematic errors, can be accounted for by adding to the astrometric equations[15] a part describing instrumental evolution based, e.g., on the best possible physical characterization of the instrumentation itself. Note that a similar scheme can be used to model residual, unknown, satellite attitude deviations from nominal, which would otherwise degrade the final astrometric error budget (Vecchiato 2012, and references therein).
It should now be clear why systems like these are named *self calibrating instruments* and, in Hipparcos, they helped overcoming technological limitations of the time. However, when unnecessary, self calibrating instruments carry the potential drawback, besides the need of substantially increasing their number, of introducing pernicious *correlations* among the astronomical and instrumental parameters utilized in the model equations describing the observations.

Active or passive designs apart, something similar to what happened with the Hipparcos mission (when technology took optical astrometry into space delivering milli-arcsec precision) must repeat itself: it is once again technology that can push space astrometry to accuracy levels unthinkable just a few years back.

5.1. Beyond Gaia

The pure passive approach adopted for the Gaia payload and its service module is allowing space astrometry to push toward the 10 μas accuracy level, 100 times better than Hipparcos. As we have seen from the considerations elaborated in the previous sections, this is certainly sufficient to open the field of *astrometric cosmology*, however it represents a serious limitation to, e.g., astrometric (i.e., direct) cosmology and fundamental physics experiments at optical wavelengths from within our SS (sec. 4.4). The use of stars or, more in general, astrophysical sources for instrument *self calibration* cannot support requirements on astrometric resolutions approaching the 1-μas level (Makarov et al. 2010 and references therein). Luckily enough, technology exists (Bertinetto & Canuto 2001; Zhai et al.

[14] The presence of extra hardware does not necessarily mean an active payload, i.e., extra moving parts and closed control loops. Calibration and monitoring measurements, for reducing systematic errors, can be taken at the same time as science observations and later reduced on the ground.

[15] These are the relativistic equations relating the satellite epoch observations to the evolution of stellar positions with time due to source finite distances and space motions, i.e., the unknown astrophysical quantities to be determined via inversion (in the least-squares sense) of the system of observation equations.

2011) that can, once put on board, disentangle fine instrumental behavior from intrinsic, and therefore astrophysically rewarding, astrometric changes of the program sources, thus allowing for the extra push forward!

In a far reaching conference like this GREAT workshop, with the Gaia launch fast approaching, inevitably *the very future of astrometry after Gaia* is put into question.
There is indeed a huge science gap left open by Gaia's 10-μas astrometry, e.g.:

1. (near)Earth-like extrasolar planets and their ephemeris call for truly μas (or better) single-exposure accuracy;
2. Local Cosmology will undergo a revolution if objects could be "pin pointed" to 10%, or better, of their positions and velocities up to a scale of 100 Kpc ($\sim$ 10 times further than the $\sim$ 5 kpc covered with Gaia);
3. Fundamental physics would see gravitational theories astrometrically tested and/or upper limits placed on them to unprecedented levels (e.g., PPN γ down to 10^{-8}).

Scientists and engineers will have to discuss again the tenets of space astrometry: is all-sky coverage always needed[16] (e.g. accurate reference frame)? Is a survey required (e.g. completeness)?Can differential astrometry, and therefore ultimate systematic error control, do the job or is absolute astrometry (i.e., absolute parallaxes) a necessity? Is long operations (several years) required?
In defining scientific goals, designers might find out that multiple, smaller missions might be better, i.e. more scientific rewarding, and cheaper than multipurpose larger facilities.

Finally, *are people actively thinking of an after Gaia?* Two designs for astrometric missions where submitted to ESA in response to their latest call for medium class missions! One, the GAME mission, was discussed at this meeting and is primarily dedicated to fundamental physics (see Gai, this volume; see also Gai et al. 2012); the other, NEAT, concentrates on the sub-μas astrometric characterization of Earth-like extrasolar planets (Malbet et al. 2012).

[16] To be distinguished from *all/sky accessibility* that should always be provided.

Hopefully, ESA will not wait for too long before initiating dedicated, both scientific and technological, assessment studies; and NASA will reconsider sooner than not their position on US astrometric missions, and to strengthen synergies and collaborations. That the scientific community is ready is crystal clear!

6. Conclusions

The role of astrometry has been revamped thanks to technology that has provided access to space and the possibility to implement the Gaia concept. For this, over the next decade or two we will know more of the real story of dark matter (and dark energy) and the validity of GR, i.e., we will be practicing a new branch of fundamental astronomy: *Astrometric Cosmology*!

The hope is that the actual geometry of the Universe, which astrometry might help unveiling through tests from within the SS, will regain ordinary matter, the baryons of which we are made, some of its role that the story told today by the *concordance model* (Seife 2003) assigns almost entirely to the mystery of dark matter and dark energy.

From the technological standpoint, space appears once again as the key to the next breakthrough in space astrometry, the place for: (a) producing the first "extrasolar ephemerides" of a distant planetary system SS alike, or (b) the most direct and extreme tests of matter's light bending properties to probe the validity of GR. A repetition of the 1919 experiment by Dyson, Eddington, and Davidson (1920), but this time to challenge, with 21st century technology, the last of *classical* theories, Einstein's General Relativity itself.

Acknowledgements. I am indebted to B. Bucciarelli, M. Crosta, F. de Felice, M. Gai, P. Re Fiorentin, A. Spagna, and A. Vecchiato for their precious help through the preparation of this contribution and for giving me access to material in advance of publication. My gratitude to M. Crosta for clarifying concepts that would otherwise have remained obscure to me.
Finally, I dedicate this article to Prof. P.L. Bernacca

for his forward vision and dedication that provided new impetus to modern astrometry in Italy.
Support for this work comes from the Italian Space Agency (ASI) under contract I/058/10/0.

References

Bertinetto, F. & Canuto, E. 2001, Optic. Engineer., 40(1), 76
Bertotti, B., Iess, L. & Tortora, P. 2003, *Nature*, 425, 374
Capetti, A., Macchetto, F. & Lattanzi, M. G. 1997, ApJ, 476, L67
Capozziello, S. & Faraoni, V. 2011, *Beyond Einstein Gravity*, Springer
Capozziello, S., Troisi A. 2005 Phys. Rev. D 72, 044022
Caraveo, P., et al. 1998, A&A, 329, L1
Cresci, et al. 2010, Nature, 467, 811
Crosta, M. 2011, Class. Quantum Grav., 28, 235013
Crosta, M. & Vecchiato, A. 2010, A&A509, A37
Crosta, M. & Mignard, F. 2006, Class. Quantum Grav., 23, 4853
Curir, A., et al. 2012, A&A, 545, A133
Damour, T., & Nordtvedt, K. 1993, Phys. Rev. Lett., 70, 2217
Damour, T., Piazza, F., & Veneziano, G. 2002, Phys. Rev. D, 66, 046007
Dyson, F. W., Eddington, A. S., & Davidson, C. 1920, Philo. Trans. of the Royal Society of London, 220A, 291
ESA 1997, *The Hipparcos and Tycho Catalogues*, ESA SP 1200
Eubanks, T. M. 1991, BAAS, 23, 1465
Ferreira, P. G. & Starkman, G. D. 2009, *Science*, 326, 812
Gai, M., et al. 2012, *Exp Astron* 34,165180
Gouda, N. 2011, *JASMINE*, Scholarpedia, 6(10), 12021
Klioner, S. 2003, AJ, 125, 1580
Kovalevsky, J. 1984, Space Science Reviews, 39, 1–63
Lacey, C. & Cole, S. 1993, MNRAS, 262, 627
Lasker, B. et al. 2008, AJ, 136, 735
Lattanzi, M. G., et al. 2006, *Astrometric Verification Unit*, DPAC Tech. rep., GAIA-C3-TN-INAF-ML-001-2
Lattanzi, M. G. Capetti, A. & Macchetto, F. 1997, A&A, 318, 997
Makarov, V. V., Parker, D. & Ulrich, R. K. 2010, ApJ717, 1202
Fabien, M., et al 2012, *Exp Astron* 34,385413
Monet, D. G., et al. 2003, AJ, 125, 984
Perryman M. A. C. et al. 2001, A&A, 369, 339
Re Firentin, P., et al. 2012, *Société Francais d'Astronomie et d'Astrophysique (SF2A) 2012*, S. Boissier, P. de Laverny, N. Nardetto, R. Samadi, D. Valls-Gabaud and H. Wozniak (eds)
Riess, A. et al. 1998, AJ, 116, 1009
Sawangwit, U. & Shanks, T. 2010, A&G 51 5.14
Seife, C. 2003, *Science*, 302, 2038
Shao, M. 2006, in *Advances in Stellar Interferometry*, *Proc. SPIE*, 6268, 62681Z
Shu, F. H. 1982, *The Physical Universe*, University Science Books, Mill Valley, California (USA)
Spagna, A., Lattanzi, M. G., Re Fiorentin, P. & Smart, R. L. 2010, A&A, 510, L4
Unwin, S.C., et al. 2008, *PASP*, 120, 38
Vecchiato A. 2012, *Putting Einstein to test: astrometric experiments from space, fundamental physics, and local cosmology*. In *Astrophysics*, Prof. Ibrahim Kucuk (Ed.), InTech, ISBN:978-953-51-0473-5
Vecchiato, A., et al. & Gai, M. 2003, A&A399, 337
Xiang-Qun Cui, et al. 2012, Res. Astron. Astrophys., 12, 1197
Zhai, C., Shao, M., Goullioud, R., Nemati, B. 2011, *Micro-pixel accuracy centroid displacement estimation and detector calibration*, Proc. R. Soc. A (2011). doi:10.1098/rspa.2011.0255