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Effect of core-shifts on VLBI group-delays

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Abstract. It is shown that the frequency-dependence of the location of AGN "cores" - the obvious reference points for defining source positions - is diluted when VLBI group-delay measurements are used.

Key words. Astrometry: VLBI

1. Introduction

Radio Very Long Baseline Interferometry (VLBI) provides the highest precision astrometry routinely available in astronomy. The current realization of the International Celestial Reference Frame (ICRF2: IERS/IVS 2009) is based on VLBI radio source positions with quoted precisions as small as $50\,\mu$as. The basis of VLBI high astrometric precision is shown in Fig. 1. A small shift in source position, $\Delta\theta$, gives rise to a change in geometric path difference, Δl = L $cos\theta$ $\Delta\theta$ producing an additional **delay** between the signals of

(1) $\quad \Delta\tau \quad = \quad \frac{1}{c}\,\mathrm{L}\; cos\theta\;\Delta\theta$

This can be measured very precisely using the interferometer **phase**:

(2) $\quad \Delta\phi(\nu) \quad = \quad \frac{2\pi\nu}{c}\,\mathrm{L}\; cos\theta\;\Delta\theta$

Fig. 2 illustrates this frequency-dependent phase response.

However, the phase is ambiguous modulo 2π and can only be used easily for differential astrometry over small angles where differential atmospheric contributions to the paths are small. For global astrometry over the whole sky the frequency dependence of the phase is used, which gives an unambiguous, but less precise, estimate of the **group delay**:

(3) $\quad \frac{1}{2\pi}\frac{\partial}{\partial\nu}\{\Delta\phi(\nu)\} \quad = \quad \frac{1}{c}\,\mathrm{L}\; cos\theta\;\Delta\theta \qquad (= \Delta\tau)$

2. Reference points for extended sources

Since astronomical objects are not points it is important to define a reference point to which any astrometric observations refer. VLBI ob-

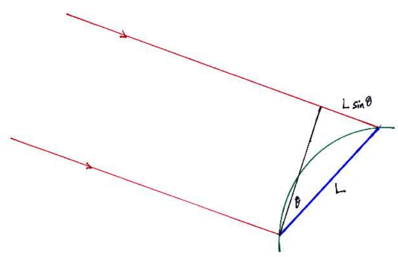

Fig. 1. Schematic interferometer. As the baseline L is large the total path difference (L $sin\theta$) is very sensitive to a small change in source position $\Delta\theta$.

Send offprint requests to: R. W. Porcas servations can easily reveal extended structures down to scales $\leq$ 1 milliarcsecond (2–3 orders of magnitude smaller than Gaia's PSF!), but the radio emission from AGNs is generally asymmetric, even on sub-beam scales, arising from a jet of relativistic plasma. Fortunately, most show a "core", a very bright, point-like feature at the end of the jet. It is possible, using this and the visible structure, to correct the interferometer phase or group delay measurements to correspond to the purely geometric response of a point source at the reference position. However, it has long been known, both observationally and theoretically, that the position of the "core" is frequency-dependent (Blandford & Königl 1979; Lobanov 1998). This **core-shift** is usually parametrized as:

$$\Delta\theta(\nu) = \Delta\theta_o + \Delta\theta_r \nu^{-\beta} \qquad (4)$$

where $\Delta\theta(\nu)$ is the core position at frequency ν, $\Delta\theta_o$ is an origin (at the base of the relativistic jet) and $\Delta\theta_r$ and β are constants.

3. Effect of core-shifts on phase and group delay

Substituting (4) in (2), the interferometer phase response of a core at frequency ν is given by:

$$\Delta\phi(\nu) = \frac{2\pi\nu}{c} \, \mathrm{L} \, cos\theta \, [\Delta\theta_o + \Delta\theta_r \nu^{-\beta}] \qquad (5)$$

Fig. 3 illustrates the phase response for 2 different positions, e.g. the jet base, $\Delta\theta_o$ and a fixed point in the jet where the core is at some arbitrary frequency.

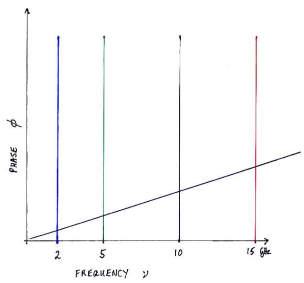

Fig. 2. Phase, ϕ, response to a small position offset. Vertical lines indicate typical radio observing bands: 2, 5, 10 & 15 GHz.

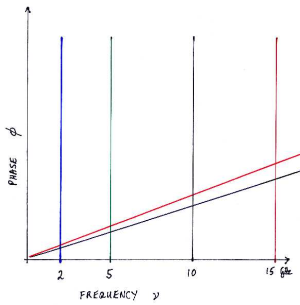

Fig. 3. Phase response versus frequency for the jet base, $\Delta\theta_o$ (lower slope) and a fixed "core" position (upper slope) further down the jet.

Substituting (4) in (3) we derive the group-delay response of a core at frequency ν:

$$\Delta\tau(\nu) = \frac{1}{2\pi}\frac{\partial}{\partial\nu}\{\frac{2\pi\nu}{c} \, \mathrm{L} \, cos\theta \, [\Delta\theta_o + \Delta\theta_r \nu^{-\beta}]\} \qquad (6)$$

$$= \frac{1}{c} \, \mathrm{L} \, cos\theta \, [\Delta\theta_o + \boldsymbol{(1-\beta)} \, \Delta\theta_r \nu^{-\beta}]$$

The most interesting - perhaps surprising - thing to note is that the term describing the frequency-dependent part of the group delay is multiplied by $\boldsymbol{(1-\beta)}$. Theoretical considerations of frequency-dependent radiation opacity at the base of a relativistic jet - the cause of the core-shift - predict that values of β should range between 0 and 1, depending on jet conditions (Lobanov (1998); β corresponds to $1/k_r$ in that paper). For this range of β the frequency-dependent term of any group-delay position measurement is *diluted* (Porcas 2009).

In the particular case of equipartition of energy between magnetic field and relativistic particle densities, and a conical jet, β is expected to be 1. Many measurements of core-shifts are indeed consistent with this value (Sokolovsky et al. 2011). For such sources the frequency-dependent term vanishes, and the position measured using group delays is that of the jet base, $\Delta\theta_o$, *at all frequencies*.

It is interesting to visualize how this arises. Fig. 4 is an elaboration of Fig. 3, showing the phase response from the jet base and a number of different positions down the jet (sketched at the right hand side). The separations of the

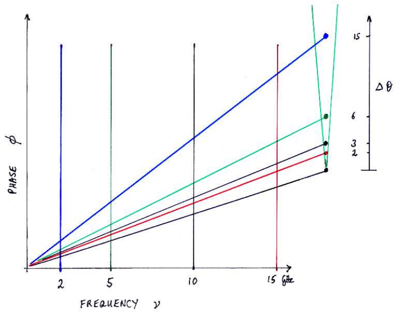

Fig. 4. Phase response versus frequency for the jet base (lowest slope) and many different positions corresponding to the points marked in the jet (sketched on right).

points from the base are proportional to the wavelengths of the 4 frequency bands shown by the vertical lines, mimicking the relative positions of the cores for a source with $\beta = 1$.

Now we investigate the behaviour of the phase for a frequency-dependent core position. Clearly, the phase at each frequency band is that given by the intercept of the slope for that position with the vertical line at that frequency, indicated by a point on those lines in Fig. 5.

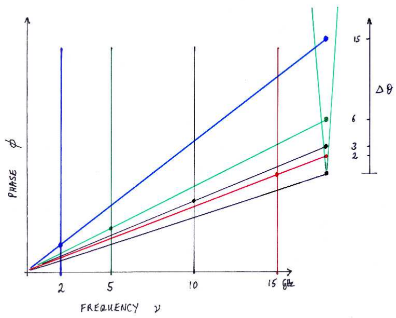

Fig. 5. Phase response to a frequency-dependent core position, given by the points of intersection of the vertical frequency lines with the corresponding slopes.

Finally, in Fig. 6 the intersection points are joined by a (rather faint) dashed line which describes the phase as a function of frequency for

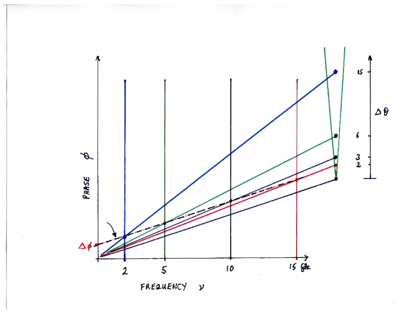

Fig. 6. Phase response with frequency for a chromatic core with $\beta = 1$, given by the faint dashed line.

a "chromatic" core. Note that, for $\beta = 1$, this line is parallel with that for the jet base; it has the same group delay. Note, also, that it does not intersect the phase axis at zero.

4. The magnitude of core-shifts

A core-shift was first established by Marcaide & Shapiro (1984) in the quasar 1038+528A, using a VLBI differential phase measurement with respect to the nearby quasar 1038+528B. They found an offset of 0.7 mas between the 2.3 and 8.4 GHz core positions, a small fraction of the 2.3 GHz beamwidth (PSF). It is possible that this core-shift is overestimated due to beam blending – see Porcas & Rioja (1997). However, Kovalev et al. (2008) found similar values for a sample of 29 sources, with a median value of 440 μas, using secondary peaks in the source jets as (presumed achromatic) reference points.

Marcaide et al. (1985) estimated a value for the exponent, β, between 0.7 and 2 for 1038+52A. Sokolovsky et al. (2011) have analysed VLBA images of a sample of 20 sources observed at a range of frequencies between 1.4 and 15.4 GHz, and found values of β consistent with 1. Taken all together these results imply a typical core-shift at wavelength λ(cm) of $\sim$50 μas/cm. Note that for a given VLBI array the beamwidth also scales with wavelength; for the VLBA it is $\sim$250 μas/cm, and hence the core-shift is only $\frac{1}{5}$th of the beamwidth *at any frequency*.

5. Some consequences

VLBI positions for over 3000 radio sources have been measured to date at 8.4 GHz using group delays, many with precisions as good as 40 μas (see Petrov et al. (2008) and references therein). Some 295 of these positions, selected on the basis of their stability and lack of significant associated extended source structure, define the present ICRF2. This quasi-inertial reference frame provides an exquisitely precise backdrop against which the Earth's rotation can be studied. Paradoxically, however, since the source core-shift parameters are unknown, we cannot locate these precise positions in the 8.4 GHz radio-emitting sky to better than ~200 μas! Locating these mathematical fiducial points within the sources needs as much attention as locating telescope "phase center" reference points within the Terrestrial Reference Frame.

One consequence of this is that ICFR2 positions cannot be used directly in phase-reference astrometry for location of astronomical sources - or spacecraft - within the ICRF without a determination of source core-shift parameters. In the past this has not been an important issue since the measurement precision of source positions was comparable with the shifts. However, recent improvements in VLBI observational techniques, and the prospect of even better precision in the future (Porcas 2010) will make this necessary. The Gaia mission should result in a large number of optical positions for quasars with precisions as good as, or exceeding, those of VLBI positions. Alignment of the Gaia- and VLBI-based celestial reference frames will depend on an understanding of the locations of both radio and optical positions within quasars; indeed, the exercise should provide some interesting astrophysical results. If most quasar cores do, in fact, have core-shifts with values of β close to 1 then the VLBI positions represent points much closer to the central black-hole and its accretion disk than the frequency-dependent radio-emitting "core" positions themselves.

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